Multi-period Optimal Procurement and Demand Responses in the Presence of Uncertain Supply

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Abstract— **We propose a simple model that integrates two-period electricity markets, uncertainty in renewable generation, and real-time dynamic demand response. A load-serving entity decides its day-ahead procurement to optimize expected social welfare a day before energy delivery. At delivery time when renewable generation is realized, it sets prices to manage demand and purchase additional power on the real-time market, if necessary, to balance supply and demand. We derive the optimal day-ahead decision, propose real-time demand response algorithm, and study the effect of volume and variability of renewable generation on these optimal decisions and on social welfare.**

I. Introduction

Future Smart Grid involves changes in both the demand side and supply side. On the supply side, more renewable energy will be integrated to reduce greenhouse gas emissions and other pollution. On the demand side, smarter demand management systems will be available to respond to the electricity price and improve the efficiency of consumption.

One of the key challenges in integrating renewable energies such as wind and solar is their uncertainty. Furthermore, there is also randomness in user demand. Despite the uncertainties, the supply and demand need to be matched at any time due to the high cost of energy storage.

On the supply side, the current mechanism for matching the demand is to have multiple periods of energy procurement: day-ahead, hour-ahead, and minutes-ahead, etc. When the delivery time approaches, one has better estimations of the supply and demand (i.e., the uncertainties decrease), but the cost of energy supply increases [8]. For simplicity, in this paper we assume two-period procurement: day-ahead and real-time. Specifically, an amount of energy generation is scheduled one day ahead for each time period in the next day, based on estimations of the renewable-energy supply and user demand at that time. Then, in real time, based on the actual demand and the available renewable energy, an amount of balancing energy is procured to meet the demand.

On the other hand, demand responses of the users will play an increasingly important role in matching the supply. Smart appliances and demand management systems will enable users to reduce their demand when the energy is expensive to produce (e.g, during peak hours) or shift the demand to other times when the supply is abundant (e.g., when the renewable energy generation is high). This is particularly useful for appliances whose demand is elastic, such as electric vehicles, air conditioners, dish washers, etc.

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In this paper, we propose a simple model that integrates two-period electricity markets, uncertainty in renewable generation, and real-time dynamic demand response. To maximize the social welfare, we derive the optimal day-ahead decision by the load-serving entity, and propose real-time demand response algorithm for the users. Furthermore, we study the effect of the volume and variability of renewable generation on these optimal decisions and on social welfare.

A. Related works

Although there is a large literature on demand response, there are relatively few works that study demand response with uncertain supply of renewable energy and uncertain demand, and consider the cost/utility structures of both the supply side and demand side in an integrated way. For example, a group of works deal with modeling and controlling specific appliances. [4] and [5] consider the electricity load control with thermal mass in buildings; [6] considers the effect of charging electric vehicles on the power distribution grid. These works focus on the demand side technologies. Another group of works consider scheduling and coordination of different appliances in households through mathematical programming, either to balance the electricity bill and waiting time [7], or maximize the social welfare [3]. The approach in [7] is based on price prediction, without considering how the price is decided. In [3], the electricity generation cost is abstracted into a simple convex function. Also, none of the above papers consider the incorporation of renewable energy on the supply side, randomness on the demand side, and their effects on demand response algorithms and the resulting social welfare.

Reference [8] considered optimal multi-period energy procurement in the presence of wind energy, and also described a demand response scheme based on "interruptible power contracts". This is different from the demand response schemes considered here where users react to the cost signals from the load-serving entity. Reference [9] considered the problem of choosing electricity prices to maximize the profit of the system operator, in the presence of renewable energy and demand response. Our objective in this paper is to maximize social welfare.

The rest of the paper is organized as follows. Section II describes the model. In Section III and IV, we design decentralized energy procurement and demand response algorithms with uncertainties in the renewable energy and demand. We start with the simpler case without time correlation (Section III) and then the case with time correlation (Section IV). Section V studies how the statistics of renewable energy affects the social welfare. Numerical results are provided in Section VI, and concluding remarks are made in Section VII.

II. Model

Our model tries to capture three different aspects of the power system. First, the load-serving entity (or LSE) procures power in two wholesale markets, the day-ahead market and real-time balancing market. It has to decide how much power to procure for each of the consumption periods (15, 30, or 60 mins in length) in the day-ahead market in advance of the consumption (e.g., by noon of the previous day), in the presence of uncertainty in supply and demand. When the random supply and demand are realized at (right before) delivery time, the LSE has to procure power in the real-time market to balance supply and demand, likely at a higher price. Second, the chief uncertainty in supply comes from renewable generation such as wind and solar, i.e., we ignore contingency events such as generator or line outages. Third, not only is the load uncertain, it will also be actively managed through demand response in real time. In this paper, we propose a simple model and an approach to demand response that integrates these three aspects. We assume that the LSE is regulated such that her objective is not to maximize her revenue by selling electricity. Instead, she cooperates with the users to maximize the social welfare (i.e., the total user utility minus the cost of providing electricity.)

Supply model:

Our model only includes the day-ahead energy market and the real-time balancing energy market; it does not cover ancillary services such as regulation and reserves. We model separately renewable power generation, such as wind and solar, because they are non-dispatchable. Specifically, consider a LSE, N users and T time periods $t = 0, 1, \ldots, T-1$. For example, these T time periods may span a day, and each period t can last 15, 30, or 60 mins, representing the time resolution for control (dispatch or demand response). We assume the utility can procure three types of power for delivery in each period:

- $P_d(t)$: a nonnegative control variable representing the day-ahead dispatchable power scheduled (or reserved) for period t. Scheduling $P_d(t)$ incurs a cost of $c_d(P_d(t);t)$. Depending on the actual demand in period t, $P_o(t) \le P_d(t)$ is consumed, which incurs an additional operation cost $c_o(P_o(t);t)$.
- $P_r(t)$: a nonnegative random variable representing the non-dispatchable renewable power that is realized at the beginning of period t. It incurs a cost of $c_r(P_r(t);t)$.
- $P_b(t)$: a nonnegative quantity representing the power the utility will buy in the real-time market to balance supply and demand for period t. It incurs a cost of $c_b(P_b(t);t)$.[1]

Assume that $c_d(\cdot;t), c_o(\cdot;t), c_b(\cdot;t)$ and $c_r(\cdot;t)$ are increasing and convex, and satisfy $c_d(0;t) = c_o(0;t) = c_b(0;t) = c_r(0;t) = 0$. It is desirable to use as much renewable power as possible; for notational simplicity only, we will assume $c_r(P;t) = 0$ for all $P \ge 0$ and all t.

[1]In practice, the LSE may also procure regulation for power balance.

User model:

Without loss of generality, we assume each user i has one appliance (e.g., an electric vehicle)[2]. During the day, user i uses the appliance (denoted by $\delta_i = 1$) with probability π_i, independent of the renewable generation P_r. In period t, user i obtains a utility of $u_i(x_i(t);t)$ from the consumption of $x_i(t)$ amount of power. Hence the random demand from user i at t is $x_i(t)\delta_i$ where δ_i is a random variable taking value 1 with probability π_i and 0 with probability $1-\pi_i$. The aggregate demand at time t is $D(t) := \sum_i x_i(t)\delta_i$. User i's consumption requirements (or constraints) are

$$\underline{x}_i(t) \le x_i(t) \le \overline{x}_i(t), \quad (1)$$

$$\sum_{t=0}^{T-1} x_i(t) \ge x_{i,total}. \quad (2)$$

For instance, if user i's electric vehicle must be charged in a subset of time periods $\mathscr{S}_i$ (e.g., 7pm to 7am), then $\underline{x}_i(\tau) = \overline{x}_i(\tau) = 0$ for $\tau \notin \mathscr{S}_i$. (We also write $\delta_i(t) := \delta_i I(t \in \mathscr{S}_i)$. Then $D(t) = \sum_i x_i(t)\delta_i(t)$.) This model is quite general and can be used for electric vehicles, dish washers, clothes washers, etc. Other reference, such as [3], model the utility functions and consumption constraints of other types of appliances, such as air-conditioners, lightings, and entertainment systems. Note that our framework can easily accommodate constraints other than (1) and (2), such as those in [3].

An alternative model of the user utility is as follows. Let $y_i(t), t = 0, 1, \ldots, T-1$ be the target consumption of user i in time period t (which have aggregated the target consumptions of all his appliances). Let his total utility function be

$$\sum_t u_i(x_i(t);t) = -\sum_t [x_i(t) - y_i(t)]^2 \quad (3)$$

which measures how much his actual consumptions differ from the target. (More changes or shifts of his demand leads to a lower value of the utility function.) This model can be viewed as a special case of the previous model by defining $u_i(x_i(t),t) = -(x_i(t) - y_i(t))^2$ and $\delta_i = 1$ for all i.

System model:

The system operates as follows. By a certain time (say, noon) of the day before delivery, the LSE decides the amount $P_d := (P_d(t), t = 0, \ldots, T-1)$ of power to purchase on the day-ahead market for each period t in the following day. At this decision time, the information available to the LSE is the probability distribution of renewable production $P_r := (P_r(t), t = 0, \ldots, T-1)$ in the next day; and the information available to the users includes the utility functions u_i and the distribution of δ_i. By time $t-$, we assume that the renewable-energy generation $P_r(t)$ and the random variables $\delta_i(t)$ have been realized, and the users have decided their demands $x_i(t), \forall i$. (Throughout the paper, user i's decisions $x_i(t)$ are made by his automated demand management system on his behalf.) With the aggregate demand $\sum_{i \in I(t)} x_i(t)$, where $I(t)$ is the set of users i with $\delta_i(t) = 1$, the LSE will purchase $P_b(t) = [\sum_{i \in I(t)} x_i(t) - P_r(t) - P_d(t)]_+$ on the real-time market to balance supply and demand.

[2]Alternatively, we can also use i to index appliances, which does not affect our results.

Given the demand vector $x(t) := (x_i(t), \forall i)$, denote by $\Delta(x(t)) := \sum_i x_i(t)\delta_i(t) - P_r(t)$ the *excess demand*, in excess of the random renewable generation $P_r(t)$. Then the total cost of the utility for each period t of the next day is a random variable given by

$$c(x(t), P_d(t); \delta(t), P_r(t), t) := \begin{cases} c_d(P_d(t);t) & \text{if } \Delta(x(t)) \le 0 \\ c_d(P_d(t);t) + c_o(\Delta(x(t));t) & \text{if } 0 < \Delta(x(t)) \le P_d(t) \\ c_d(P_d(t);t) + c_o(P_d(t);t) + & \\ \quad c_b(\Delta(x(t)) - P_d(t);t) & \text{if } P_d(t) < \Delta(x(t)) \end{cases} \tag{4}$$

where $\delta(t)$ denotes the vector $(\delta_i(t), \forall i)$. Equivalently,

$$c(x(t), P_d(t); \delta(t), P_r(t), t) = c_d(P_d(t);t) + c_o((\Delta(x(t)))_0^{P_d(t)};t) + c_b((\Delta(x(t)) - P_d(t))_+;t) \tag{5}$$

where $(z)_a^b := \min\{\max\{z,a\},b\}$; $(z)_+ := z$ if $z \ge 0$ and 0 if $z < 0$. For convenience, we will write $c(x(t), P_d(t); \delta(t), P_r(t), t)$ as $c(x(t), P_d(t); t)$. The social welfare for the next day is defined as

$$W_{total}(x, P_d) = \sum_{t=0}^{T-1} \{\sum_i [\delta_i(t) u_i(x_i(t);t)] - c(x(t), P_d(t);t)\} \tag{6}$$

In this paper, we investigate the optimal energy procurement and demand response schemes of the LSE and the users. Note that the cost structure of the LSE and the utility structure of the users are both private. So the ideal schemes should only rely on limited information exchange without requiring the users know the cost functions of the LSE, and vice versa.

III. The Case without Time Correlation

To gain some intuition, we first consider the simpler case without the time-correlation constraint (2). Therefore, maximizing the expected social welfare of the day $E[W_{total}]$ reduces to maximizing the expected social welfare for each time period. Denote the social welfare in period t by

$$\begin{aligned} & W(x(t), P_d(t); \delta(t), P_r(t), t) \\ := \; & \sum_i [\delta_i(t) u_i(x_i(t);t)] - c(x(t), P_d(t);t). \end{aligned} \tag{7}$$

For convenience, we will drop the notation t, and simply write $W(x(t), P_d(t); \delta(t), P_r(t), t)$ as $W(x, P_d; \delta, P_r)$ or $W(x, P_d)$. We will make the following assumptions throughout the paper:

A0: First, all utility functions u_i are concave and satisfies $|u_i'(x_i(t);t)| < V < \infty, \forall x_i(t) \in [\underline{x}_i(t), \bar{x}_i(t)], \forall t$; the cost functions $c_d(\cdot)$, $c_o(\cdot)$, and $c_b(\cdot)$ are convex and increasing with $c_d(0) = c_o(0) = c_b(0) = 0$. Second, $c_b'(0) > c_o'(P), \forall P \ge 0$. That is, the marginal cost of real-time energy is strictly more than the marginal operation cost of day-ahead energy. Finally, assume that there exists $P_{max} \ge \sum_i \bar{x}_i$ such that $c_d'(P_{max}), c_o'(P_{max}), c_b'(P_{max}) < B < \infty$. This implies that one can support the maximal possible demand $\sum_i \bar{x}_i$ using only day-ahead energy or real-time energy with finite marginal cost.

A. Day-ahead pricing/planning

One possible scheme is as follows. One day ahead, the LSE sets the prices to be imposed on the users for each time period of the next day, and also decides the quantities $P_d(t)$. According to the prices, each user i optimizes his energy consumption in the event that he intends to use the appliance. Under this scheme, the prices are determined one day in advance and are therefore *predictable* to the users. So, user i's demand $x_i(t)$, in the event that he is active, is also predictable (but note that $x_i(t)$'s are generally different for different period t's). Therefore, we can view $x_i(t)$ as being determined one day in advance. However, his actual demand $x_i(t)\delta_i(t)$ is random due to the randomness of $\delta_i(t)$. The renewable energy $P_r(t)$'s are also random.

We would like to design an energy procurement and demand response scheme to find x and P_d (for each time period t) that solve the following problem:

$$\begin{aligned} \max_{x, P_d} \quad & EW(x, P_d) \\ \text{subject to} \quad & \underline{x}_i \le x_i \le \bar{x}_i \quad \text{and} \quad P_d \ge 0 \end{aligned}$$

where $EW(x, P_d)$ is the expectation of $W(x, P_d; \delta, P_r)$, taken over the distribution of δ and P_r. Later, the prices charged by the LSE will naturally appear from this formulation.

First, we have the following property.

Proposition 1: $EW(x, P_d)$ is concave in (x, P_d).

Proof: Given P_d, x, δ, and P_r, consider the following convex optimization problem:

$$\begin{aligned} F(x, P_b) := \quad \min_{y_o, y_b} \quad & c_o(y_o) + c_b(y_b) \\ s.t. \quad & 0 \le y_o \le P_b, y_b \ge 0, \\ & P_r + y_o + y_b \ge \sum_i \delta_i x_i, \end{aligned} \tag{8}$$

where y_o is the amount of energy in P_d that is used, and y_b is the amount of balancing energy. By assumption A0, $c_b'(0) > c_o'(P), \forall P \ge 0$. So, in the optimal solution, one uses the reserved day-ahead energy before resorting to the real-time balancing energy. Therefore the optimal solution is that $y_o^* = (\sum_i \delta_i x_i - P_r)_0^{P_d}$ and $y_b^* = (\sum_i \delta_i x_i - P_r - P_d)_+$. Consequently,

$$F(x, P_b) + c_d(P_d) = c_o(y_o^*) + c_b(y_b^*) + c_d(P_d) = c(x, P_d). \tag{9}$$

Note that x and P_b appear in the constraints of (8) and all constraints are linear inequalities, so $F(x, P_b)$ is convex in (x, P_b) [10]. By (9), $c(x, P_d)$ is convex in (x, P_d). So $W(x, P_d) = \sum_i \delta_i u_i(x_i) - c(x, P_d)$ is concave in (x, P_d). Since this holds for any δ and P_r, $E(W(x, P_d))$ is also concave in (x, P_d). ∎

We have

$$\frac{\partial c}{\partial P_d}(x, P_d; \delta, P_r) = c_d'(P_d) + I(\Delta(x) \ge P_d) \cdot [c_o'(P_d) - c_b'((\Delta(x) - P_d)_+)] \tag{10}$$

$$\frac{\partial c}{\partial x_i}(x, P_d; \delta, P_r) = \delta_i [I(0 \le \Delta(x) < P_d) c_o'(\Delta(x)) + I(\Delta(x) \ge P_d) c_b'((\Delta(x) - P_d)_+)]. \tag{11}$$

So,

$$\frac{\partial EW}{\partial P_d}(x,P_d) = -E[\frac{\partial c}{\partial P_d}(x,P_d;\delta,P_r)] \quad (12)$$

$$\frac{\partial EW}{\partial x_i}(x,P_d) = u_i'(x_i)\pi_i - E[\frac{\partial c}{\partial x_i}(x,P_d;\delta,P_r)] \quad (13)$$

Strictly speaking, generally $\frac{\partial EW}{\partial P_d}(x,P_d)$ is not continuous at the point $P_d = \Delta(x)$, and $\frac{\partial EW}{\partial x_i}(x,P_d)$ is not continuous at the points $\Delta(x) = 0$ and $\Delta(x) = P_d$. However, the above expressions are still subgradients [10], and do not affect our results.

We design the following algorithm to find the optimal day-ahead decisions P_d^* and x^*. We assume that the automated demand management system of user i has knowledge of the distribution of δ_i, and the LSE knows the distribution of P_r. The algorithm is run one day ahead by simulating the system according to the distributions.

Algorithm 1: day-ahead planning

Initially, user i chooses $x_i^0 \in [\underline{x}_i, \bar{x}_i]$, and the LSE chooses $P_d^0 \in [0, P_{max}]$.

In iteration $m+1 = 1,2,\ldots$, do the following.

1) Each user i generates a sample δ_i^m, and the LSE generates a sample P_r^m.
2) Active users (i.e., those with $\delta_i^m = 1$) report x^m to the LSE through a communication network. The LSE then computes $\hat{p}^m = \hat{p}(\Delta(x^m), P_d^m)$ and reports $\hat{p}^m$ to all active users, where

$$\hat{p}(\Delta(x^m),P_d^m) := I(0 \le \Delta(x^m) < P_d^m)\cdot c_o'(\Delta(x^m)) + I(\Delta(x^m) \ge P_d^m)\cdot c_b'((\Delta(x^m) - P_d^m)_+) \quad (14)$$

with

$$\Delta(x^m) = \sum_i (\delta_i^m x_i^m) - P_r^m.$$

The LSE also updates P_d^{m+1}:

$$P_d^{m+1} = \{P_d^m - \alpha^m \frac{\partial c}{\partial P_d}(x^m, P_d^m; \delta^m, P_r^m)\}_0^{P_{max}}$$

3) User i updates x_i^{m+1}:

$$x_i^{m+1} = \left(x_i^m + \alpha^m \delta_i^m \cdot [u_i'(x_i^m) - \hat{p}^m]\right)_{\underline{x}_i}^{\bar{x}_i}$$

where $\alpha^m > 0$ is the step size.

Note that Algorithm 1 is a stochastic gradient projection algorithm [11], [12] since $E\{\delta_i \cdot [u_i'(x_i^m) - \hat{p}^m]\} = \frac{\partial EW}{\partial x_i}(x^m, P_d^m)$, and $-E[\frac{\partial c}{\partial P_d}(x^m, P_d^m; \delta^m, P_r^m)] = \frac{\partial EW}{\partial P_d}(x^m, P_d^m)$.

Proposition 2: Let $\mathscr{B}$ be the set of optimal solutions (x^*, P_d^*) to the welfare-maximization problem (8). Note that $\mathscr{B}$ is a convex set. If the step sizes satisfy $\sum_m \alpha^m = \infty$ and $\sum_m (\alpha^m)^2 < \infty$ (for example, if $\alpha^m = 1/(m+1)$), then (x_i^m, P_d^m) converges to the set $\mathscr{B}$ (i.e., $\lim_{m\to\infty} \min_{\mathbf{y}\in\mathscr{B}} ||(x^m, P_d^m)^T - \mathbf{y}||_2 = 0$) w.p. 1. By the continuity of $W(x,P_d)$, we have $E[W(x^m, P_d^m)] \to E[W(x^*, P_d^*)]$ w.p. 1.

Proof: Algorithm 1 is a stochastic gradient projection algorithm. Note that all the increments $-\frac{\partial c}{\partial P_d}(x^m, P_d^m; \delta^m, P_r^m)$ and $\delta_i^m \cdot [u_i'(x_i^m) - \hat{p}^m]$ are bounded. To see this, note that $x_i^m \in [\underline{x}_i, \bar{x}_i]$ is bounded. So $\Delta(x^m) \le P_{max}$. Using this fact together with Assumption A0, we know that (10) and (11) are bounded. By Theorem 3.1 in [12] (with $||(x_i^m, P_d^m)^T - (x^*, P_d^*)^T||_2^2$ as the Lyapunov function, where (x^*, P_d^*) is any optimal solution), (x_i^m, P_d^m) converges to the set $\mathscr{B}$ w.p. 1. ■

Remark: At an optimum point (x^*, P_d^*), if we ignore the constraint $x_i \in [\underline{x}_i, \bar{x}_i]$ for easy exposition, then $E\{\delta_i \cdot [u_i'(x_i^*) - \hat{p}^*]\} = \pi_i u_i'(x_i^*) - E(\delta_i \hat{p}^*) = 0$, where $\hat{p}^* := \hat{p}(\Delta(x^*), P_d^*)$. That is, $u_i'(x_i^*) = E(\delta_i \hat{p}^*)/\pi_i$. Therefore, if user i is charged a price of $\bar{p}_i := E(\delta_i \hat{p}^*)/\pi_i$, then he will choose to consume x_i^*. In other words, in Algorithm 1 user i implicitly takes the price $\bar{p}_i$. Note that $\bar{p}_i$ is the expected marginal cost of user i's consumption, conditioned on the event that he is active. (It can be estimated in Algorithm 1 as $\frac{\sum_{m=1}^M (\delta_i^m \hat{p}^m)}{\sum_{m=1}^M \delta_i^m}$ with a large M, which is basically the average price user i sees.) Interestingly, unlike the case without randomness in demand [3], the price $\bar{p}_i$ is generally different for different users. (However, when the system is large, we conjecture that $E(\delta_i \hat{p}^*)/\pi_i \approx E(\hat{p}^*)$.)

B. Real-time demand response

Now we consider another scheme where the price and user demand are determined in real time based on the realization of δ and P_r (instead of the statistics of δ and P_r in the previous scheme). Note that P_d still needs to be chosen one day in advance, based on the statistics of δ and P_r. Again, we focus on one time period t.

The real-time consumption decisions $x(t) := (x_i(t), i = 1,\ldots,N)$ are made at the beginning of each period t. At time $t-$, the day-ahead power $P_d(t)$ has been committed, and the set $I(t) := \{i \mid \delta_i(t) = 1\}$ of appliance users and the renewable-energy production $P_r(t)$ have been realized. So at time t, the real-time social welfare $W(x(t), P_d(t); \delta(t), P_r(t), t)$, or simply written as $W(x, P_d; \delta, P_r)$, is no longer random. Our problem is to determine

(i) the optimal real-time consumption $x_i(P_d; \delta, P_r)$ which depends on P_d and the realization of δ, P_r;

(ii) the optimal day-ahead energy P_d that solves

$$\max_{P_d \ge 0} E[W(x(P_d;\delta,P_r), P_d; \delta, P_r)]$$

where the expectation is taken over δ and P_r.

For (i), since P_d has been committed, the cost $c_d(P_d)$ has been given. So, the LSE and the users need to solve

$$\tilde{W}(P_d;\delta,P_r) := \max_{x\in[\underline{x},\bar{x}]} \{\sum_i (\delta_i u_i(x_i)) - c_o((\sum_i (\delta_i x_i) - P_r)_0^{P_d}) - c_b((\sum_i (\delta_i x_i) - P_r - P_d)_+)\} \quad (15)$$

which is equivalent to

$$\begin{aligned} \tilde{W}(P_d;\delta,P_r) := & \max_{x,y_o,y_b} \{\sum_i (\delta_i u_i(x_i)) - c_o(y_o) - c_b(y_b)\} \\ s.t.\ & \underline{x}_i \le x_i \le \bar{x}_i, \forall i \\ & 0 \le y_o \le P_d, y_b \ge 0 \\ & P_r + y_o + y_b \ge \sum_i (\delta_i x_i). \end{aligned} \quad (16)$$

Associate dual variables μ_1 and μ_2 with the last two constraints. Then a partial Lagrangian is

$$\begin{aligned}&\mathscr{L}(x, y_o, y_b; \mu_1, \mu_2)\\ = &\sum_i (\delta_i u_i(x_i)) - c_o(y_o) - c_b(y_b) + \mu_1(P_d - y_o)\\ &+\mu_2(P_r + y_o + y_b - \sum_i(\delta_i x_i)). \end{aligned} \tag{17}$$

So, a primal-dual algorithm to solve problem (16) is as follows.

Algorithm 2: Given P_d, δ, P_r, compute x

Initially, for any active user i (i.e., with $\delta_i = 1$), let $x_i^0 \in [\underline{x}_i, \bar{x}_i]$. The LSE lets $\mu_1^0 = \mu_2^0 = 0$, and $y_o^0 = y_b^0 = 0$. In iteration $k+1 = 1, 2, \ldots$, do the following.

1) Each active user i computes x_i^{k+1}, and report it to the LSE through a communication network:

$$x_i^{k+1} = \left(x_i^k + \beta^k \cdot [u_i'(x_i^k) - \mu_2^k]\right)_{\underline{x}_i}^{\bar{x}_i}$$

where $\beta^k > 0$ is the step size. That is, the "price" posed to the users is μ_2^k.

2) The LSE computes $\mu_1^{k+1}, \mu_2^{k+1}, y_o^{k+1}, y_b^{k+1}$:

$$\begin{aligned}\mu_1^{k+1} &= [\mu_1^k + \beta^k(y_o^k - P_d)]_+,\\ \mu_2^{k+1} &= [\mu_2^k + \beta^k(\sum_i(\delta_i x_i^k) - P_r - y_o^k - y_b^k)]_+,\\ y_o^{k+1} &= [y_o^k + \beta^k(-c_o'(y_o^k) - \mu_1^k + \mu_2^k)]_0^{P_{max}},\\ y_b^{k+1} &= [y_b^k + \beta^k(-c_b'(y_b^k) + \mu_2^k)]_0^{P_{max}}\end{aligned}$$

where B and P_{max} are defined in Assumption A0. The LSE reports μ_2^{k+1} to active users.

With proper step sizes (e.g., $\beta^k = 1/(k+1)$), Algorithm 2 converges to the set of optimal solutions and dual variables. Denote by μ_1^* the value of μ_1^k after convergence.

Proposition 3: $\tilde{W}(P_d; \delta, P_r)$ is concave in P_d. Also,

$$\partial \tilde{W}(P_d; \delta, P_r)/\partial P_d = \mu_1^* \tag{18}$$

if $\tilde{W}(P_d; \delta, P_r)$ is differentiable at point P_d.

Proof: Since $\tilde{W}(P_d; \delta, P_r)$ is the optimal value of the convex optimization problem (16), it is concave in P_d [10].

To show (18), note that P_d is associated with the dual variable μ_1 only. Then, this result follows from the standard sensitivity analysis [10] in convex optimization. ■

For (ii), to decide P_d to maximize social welfare, the LSE needs to solve (for each t)

$$\max_{P_d \geq 0} \{E[\tilde{W}(P_d; \delta, P_r)] - c_d(P_d)\},$$

where $\tilde{W}(.)$ is defined in (15) or (16). The gradient of the objective function is $g(P_d) := E(\mu_1^*) - c_d^{'}(P_d)$ (note that μ_1^* depends on P_d, P_r, δ). So, a stochastic subgradient algorithm that converges to the set of optimal P_d is as follows. Similar to Algorithm 1, Algorithm 3 can be run one day in advance by simulating the system (i.e., drawing samples of δ and P_r).

Algorithm 3

1) Initially, let $P_d^0 = 0$.

2) In step $m+1 = 1, 2, \ldots$, given a realization of P_r and δ (denoted by P_r^m and δ^m), run Algorithm 2 to find μ_1^*, and denote it by μ_1^{*m}. Then, compute

$$P_d^{m+1} = \{P_d^m + \alpha^m[\mu_1^{*m} - c_d^{'}(P_d^m)]\}_0^{P_{max}}$$

where $\alpha^m = 1/(m+1)$ is the step size.

Remark: Algorithm 3 is run one day ahead. Step 2) of Algorithm 3 uses Algorithm 2. So Algorithm 2 is run as part of Algorithm 3 one day ahead to determine P_d. Note that Algorithm 2 is also run at time $t-$ after $\delta(t)$ and $P_r(t)$ are realized, to determine the optimal real-time consumption.

IV. The Case with Time Correlation

Now we consider the case with the time correlation constraint (2). Other constraints in, for example, [3], can be treated similarly.

A. Day-ahead pricing/planning

Consider problem (8) with the additional constraint (2). For all i, we associate a dual variable μ_i with this constraint, and obtain the Lagrangian as

$$\mathscr{L}(x, P_d; \mu) = E[W(x, P_d)] + \sum_i \mu_i (\sum_t x_i(t) - x_{i,total}).$$

Then we have the following algorithm similar to Algorithm 1 which converges to the set of optimal solutions.

Algorithm 4

Initially, user i lets $x_i^0(t) \in [\underline{x}_i, \bar{x}_i], \forall t$, and the LSE lets $P_d^0(t) \in [0, P_{max}], \forall t$.

In iteration $m+1 = 1, 2, \ldots$, do the following.

1) Each user i generates a sample δ_i^m, and the LSE generates samples $P_r^m(t), \forall t$.

2) Each active user i (i.e., with $\delta_i^m = 1$) communicates $x_i^m(t), \forall t$ to the LSE, which then computes $\hat{p}^m(t) := \hat{p}(\Delta(x^m(t), P_d^m(t))$ and reports $\hat{p}^m(t)$ to all active users, where $\hat{p}(\Delta(x^m(t), P_d^m(t))$ is defined in (14). The LSE updates $P_d^{m+1}(t), \forall t$:

$$\begin{aligned}&P_d^{m+1}(t) = \{P_d^m(t) -\\ &\quad \alpha^m \frac{\partial c}{\partial P_d}(x^m(t), P_d^m(t); \delta^m(t), P_r^m(t))\}_0^{P_{max}}\end{aligned} \tag{19}$$

where $\alpha^m = 1/(m+1)$ is the step size.

3) User i updates $x_i^{m+1}(t), \forall t$ as

$$x_i^{m+1}(t) = \left(x_i^m(t) + \alpha^m \delta_i^m \cdot [u_i'(x_i^m(t)) - \hat{p}^m(t) + \mu_i^m]\right)_{\underline{x}_i}^{\bar{x}_i} \tag{20}$$

and the dual variable as

$$\mu_i^{m+1} = \{\mu_i^m + \alpha^m[x_{i,total} - \sum_t x_i^m(t)]\}_+.$$

B. Real-time demand response

With time correlations, the optimal real-time demand response policy is the solution of a dynamic programming problem, which is hard to obtain analytically. Instead, we propose a heuristic algorithm.

Algorithm 5

1) Initially, let the remaining demand be $\tilde{x}_i(0) := x_{i,total}, \forall i$. Using Algorithm 4, determine the day-ahead energy $P_d(t), t = 0,1,2,\ldots,T-1$ as the solution of

$$\begin{aligned} \max_{x\in[\underline{x},\bar{x}],P_d} \quad & E(\sum_{\tau=0}^{T-1} W(x(\tau),P_d(\tau);\tau)) \\ s.t. \quad & \sum_{\tau=0}^{T-1} x_i(\tau) \ge x_{i,total}, \forall i. \end{aligned}$$

2) In period t $(t = 0,1,2,\ldots,T-1)$, $\{P_r(\tau), 0 \le \tau \le t\}$ have been observed by the LSE, and $\{\delta_i(\tau), 0 \le \tau \le t\}$ have been realized for user i. So, the conditional distributions of $\{P_r(\tau), \tau > t\}$ and $\{\delta_i(\tau), \tau > t\}$ are known. Also, each active user i knows his remaining demand $\tilde{x}_i(t)$. Then similar to Algorithm 4 (but without updating P_d), the users and the LSE follow an iterative algorithm to solve

$$\begin{aligned} \max_{x(\tau),\tau\ge t} \quad & \{W(x(t),P_d(t);t) + E(\sum_{\tau'=t+1}^{T-1} W(x(\tau'), \\ & P_d(\tau');\tau')|P_r(\tau),\delta_i(\tau),\forall\tau\le t,\forall i)\} \\ s.t. \quad & \underline{x}_i(\tau) \le x_i(\tau) \le \bar{x}_i(\tau), \forall i, \tau \ge t, \\ & \sum_{\tau=t}^{T-1} x_i(\tau) \ge \tilde{x}_i(t), \forall i \end{aligned}$$

The value of $x_i(t)$ in the solution is used as the demand of user i in period t.
Finally, user i sets $\tilde{x}_i(t+1) = \tilde{x}_i(t) - x_i(t)$. Repeat step 2 after incrementing t.

To further simplify computation, we make the following approximations. In particular, assume that

$$\begin{aligned} & E[W(x(\tau),P_d(\tau);\tau)] \approx \sum_i \pi_i(\tau)u_i(x_i(\tau);\tau) - \\ & c(x(\tau'),P_d(\tau);\pi,E(P_r(\tau)),\tau). \end{aligned} \tag{21}$$

Note that the approximation is made in the cost term, where we have moved the expectation to the inside of $c(.)$.

Similarly, denote $\bar{P}_r(\tau'|t) := E(P_r(\tau')|P_r(\tau),\delta_i(\tau),\forall\tau \le t,\forall i)$, and $\bar{\delta}_j(\tau'|t) := E(\delta_j(\tau')|P_r(\tau),\delta_i(\tau),\forall\tau\le t,\forall i)$. If P_r and δ are independent and δ_i's are independent of each other, then $\bar{P}_r(\tau'|t) = E(P_r(\tau')|P_r(\tau),\forall\tau\le t)$ and $\pi_j(\tau'|t) = E(\delta_j(\tau')|\delta_j(\tau),\forall\tau\le t)$. We make the approximation that

$$\begin{aligned} & E[W(x(\tau'),P_d(\tau');\tau')|P_r(\tau),\delta_i(\tau),\forall\tau\le t,\forall i)] \\ \approx \quad & \sum_i \pi_i(\tau'|t)u_i(x_i(\tau');\tau') - \\ & c(x(\tau'),P_d(\tau');\pi(\tau'|t),\bar{P}_r(\tau'|t),\tau'). \end{aligned} \tag{22}$$

After these approximations, at each time t Algorithm 5 solves a simpler convex optimization problem.

In the future, we are interested to obtain analytical performance bounds of Algorithm 5 compared to the dynamic-programming solution. However, the simulation results in Section VI show very good performance of Algorithm 5 (with approximations (21) and (22)).

V. Effect of renewable energy on social welfare

An important element in our model is the uncertain renewable energy. In the future, the penetration of renewable energy and its impact are expected to increase. In this section, we investigate how the statistics of the renewable energy affects the social welfare in our model. For simplicity, consider the case without time correlation. So we can focus on one time period. We will find that similar results hold in both the day-ahead pricing/planning scheme and the real-time demand response scheme.

Recall that the cost function (after dropping t) is

$$c(x,P_d;\delta,P_r) = c_d(P_d) + c_o((\Delta(x))_0^{P_d}) + c_b((\Delta(x)-P_d)_+) \tag{23}$$

where $\Delta(x) = \sum_i \delta_i x_i - P_r$. Assume that δ and P_r are independent. The following can be shown similarly to Prop. 1.

Proposition 4: Given x, P_d and δ, $c(x,P_d;\delta,P_r)$ is convex in P_r.

A. Day-ahead pricing/planning

1) Effect of the variance of renewable energy: First we study how the variance of P_r affects the social welfare $E[W(x,P_d;\delta,P_r)] = \sum_i \pi_i u_i(x_i) - E[c(x,P_d;\delta,P_r)]$. Assume that the renewable energy is parametrized by $a,b > 0$ as follows:

$$P_r(a,b) := a\cdot\mu_r + b\cdot V_r$$

where $\mu_r > 0$ is a constant, and V_r is a zero-mean random variable with $E(V_r^2) = \sigma^2$. So $E[P_r(a,b)] = a\cdot\mu_r$.

Lemma 1: Given any x, P_d, and $a > 0$, if $b_2 > b_1 > 0$, then $E[c(x,P_d;\delta,P_r(a,b_2))] \ge E[c(x,P_d;\delta,P_r(a,b_1))]$. That is, when the mean of renewable energy is fixed, a larger variance leads to a higher cost.

Proof: Given x, P_d and δ, $c(x,P_d;\delta,P_r)$ is convex in P_r. In the proof, we write $c(P_r) = c(x,P_d;\delta,P_r)$ for simplicity.

Since $c(P_r)$ is convex, we have, for any v,

$$c(a\mu_r + b_1 v) - c(a\mu_r) \le c'(a\mu_r + b_1 v)\cdot(b_1 v), \tag{24}$$
$$c(a\mu_r + b_2 v) - c(a\mu_r + b_1 v) \ge c'(a\mu_r + b_1 v)\cdot(b_2 - b_1)v. \tag{25}$$

Since $b_2 > b_1 > 0$, (24) implies that

$$c'(a\mu_r + b_1 v)\cdot(b_2-b_1)v \ge \frac{b_2-b_1}{b_1}[c(a\mu_r+b_1 v) - c(a\mu_r)].$$

Combined with (25), we have

$$c(a\mu_r+b_2 v) - c(a\mu_r+b_1 v) \ge \frac{b_2-b_1}{b_1}[c(a\mu_r+b_1 v) - c(a\mu_r)],$$

which is equivalent to

$$c(a\mu_r+b_2 v) - c(a\mu_r) \ge \frac{b_2}{b_1}[c(a\mu_r+b_1 v) - c(a\mu_r)]. \tag{26}$$

Changing v to V_r, and taking expectation on both sides, we have

$$E(c(a\mu_r+b_2 V_r)) - c(a\mu_r) \ge \frac{b_2}{b_1}[E(c(a\mu_r+b_1 V_r)) - c(a\mu_r)].$$

The right-hand-side of the inequality is nonnegative since $E(c(a\mu_r + b_1 V_r)) - c(a\mu_r) \geq c(E(a\mu_r + b_1 V_r)) - c(a\mu_r) = 0$ by Jensen's inequality. Therefore,

$$\begin{aligned} & E(c(a\mu_r + b_2 V_r)) - c(a\mu_r) \\ \geq\ & \frac{b_2}{b_1}[E(c(a\mu_r + b_1 V_r)) - c(a\mu_r)] \\ \geq\ & E(c(a\mu_r + b_1 V_r)) - c(a\mu_r), \end{aligned} \quad (27)$$

which implies that $E(c(a\mu_r + b_2 V_r)) \geq E(c(a\mu_r + b_1 V_r))$. ■

Proposition 5: If $b_2 > b_1 > 0$, then

$$\max_{x,P_d} E[W(x,P_d;\delta,P_r(a,b_2)] \leq \max_{x,P_d} E[W(x,P_d;\delta,P_r(a,b_1)].$$

Proof: Given any x, P_d, Lemma 1 implies that $E[W(x,P_d;\delta,P_r(a,b_2)] \leq E[W(x,P_d;\delta,P_r(a,b_1)]$. Then the result follows. ■

2) Size of the renewable energy farm: Now we consider the effect of the size of the renewable energy farm (e.g., wind farm) on the social welfare. Let the random variable $R := \mu_r + V_r \geq 0$ be the renewable energy generated by a unit of farm capacity. Then if the farm has a size of s, then the renewable energy $P_r = s \cdot (\mu_r + V_r) = P_r(s,s)$.

Proposition 6: If $s_2 > s_1 > 0$, then

$$\max_{x,P_d} E[W(x,P_d;\delta,P_r(s_2,s_2)] \geq \max_{x,P_d} E[W(x,P_d;\delta,P_r(s_1,s_1)].$$

Proof: Given any x, P_d and δ, the cost function (23) is non-increasing in P_r. Since $s_2 R \geq s_1 R$, we have $c(x,\delta,P_d;P_r(s_2,s_2)) \leq c(x,P_d;\delta,P_r(s_1,s_1))$. So $E[W(x,P_d;\delta,P_r(s_2,s_2))] \geq E[W(x,P_d;\delta,P_r(s_1,s_1))]$. The result follows. ■

This proposition means that the social welfare increases with the farm size, despite larger variance. Note that this result is derived with the assumption that the cost of renewable energy is 0.

The following result can be proved similarly.

Proposition 7: When $b > 0$ is fixed, $\max_{x,P_d} E[W(x,P_d;\delta,P_r(a,b))]$ is a non-decreasing function of a. That is, when the variance is fixed, the social welfare is non-decreasing with the mean.

3) Effect of aggregating renewable energy from independent farms: Consider n renewable energy farms. Farm j generates energy $P_{r,j} := \mu_r + V_{r,j}$. Assume that $P_{r,j}, j = 1,\ldots,n$ are independent, and that $V_{r,j}$ is zero-mean and satisfies $E(V_{r,j}^2) = \sigma^2$. Then, when n is large, the aggregate energy $P_r = \sum_{j=1}^n P_{r,j}$ approximately follows a Gaussian distribution and can be written as $P_r := n\mu_r + \sqrt{n} W_r$ where $W_r \sim N(0,\sigma^2)$.

Proposition 8: If $n_2 > n_1 > 0$, then

$$\begin{aligned} & \max_{x,P_d} E[W(x,P_d;\delta,n_2\mu_r + \sqrt{n_2}W_r] \\ \geq\ & \max_{x,P_d} E[W(x,P_d;\delta,n_1\mu_r + \sqrt{n_1}W_r]. \end{aligned} \quad (28)$$

Proof: By Prop. 6, we have

$$\begin{aligned} & \max_{x,P_d} E[W(x,P_d;\delta,n_2\mu_r + \sqrt{n_2}W_r] \\ \geq\ & \max_{x,P_d} E[W(x,P_d;\delta,n_1\mu_r + (n_1/n_2)\sqrt{n_2}W_r]. \end{aligned} \quad (29)$$

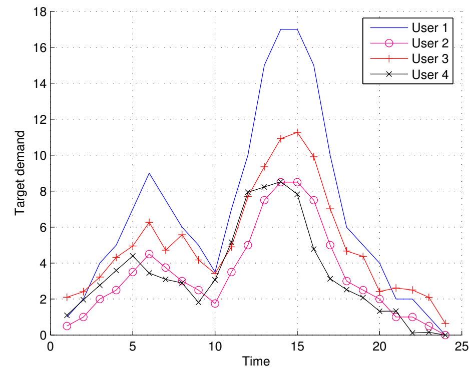


Fig. 1: Target demand profiles of the users

Since $n_2 > n_1$, it follows that $(n_1/n_2)\sqrt{n_2} \leq \sqrt{n_1}$. Then by Prop. 5, we have

$$\begin{aligned} & \max_{x,P_d} E[W(x,P_d;\delta,n_1\mu_r + (n_1/n_2)\sqrt{n_2}W_r] \\ \geq\ & \max_{x,P_d} E[W(x,P_d;\delta,n_1\mu_r + \sqrt{n_1}W_r]. \end{aligned} \quad (30)$$

Combining the two inequalities completes the proof. ■

B. Real-time demand response

In this case, the consumption x depends on P_d and the realization of P_r, δ. So, the cost function is $c(x(P_d;P_r,\delta),P_d;P_r,\delta)$.

Proposition 9: Given P_d, δ, $\tilde{W}(P_d;\delta,P_r) = W(x(P_d;P_r,\delta),P_d;\delta,P_r)$ is concave in P_r.

Proof: $\tilde{W}(P_d;\delta,P_r)$ is the optimal value of the convex optimization problem (16). Also, P_r is in the constraint. So, $\tilde{W}(P_d;\delta,P_r)$ is concave in P_r [10]. ■

Proposition 10: Similar to the case of day-ahead planning, Prop. 5, 6, 7, and 8 all hold after changing $\max_{x,P_d} E[W(x,P_d;\delta,P_r(\cdot,\cdot))]$ to $\max_{P_d}\{E[\tilde{W}(P_d;\delta,P_r(\cdot,\cdot))] - c_d(P_d)\}$.

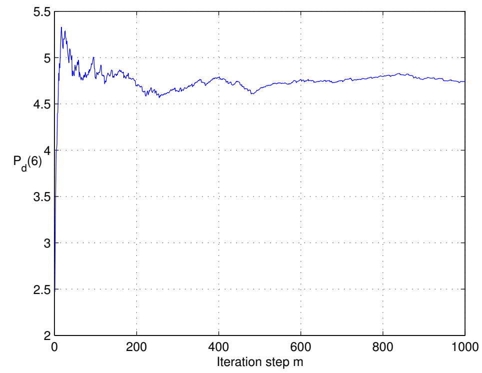
Proof: Similar to the last subsection, this is proved using the fact that $\tilde{W}(P_d;\delta,P_r) - c_d(P_d)$ is concave and non-decreasing in P_r. ■

VI. Numerical Examples

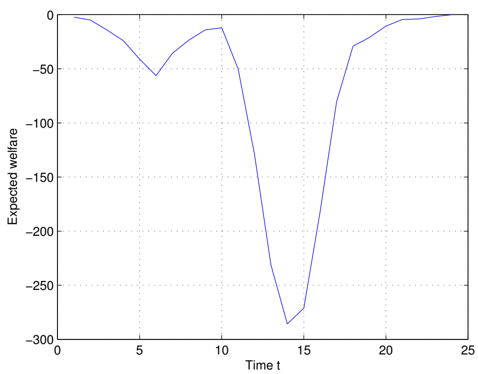
The setup is as follows. Let $T = 24$, representing 24 hours. The first time period is 8-9am, the second is 9-10am, and so on. The utility function of user i is (3). That is, the further his actual demand profile $\{x_i(t)\}$ deviates from his target demand profile $\{y_i(t)\}$, the less is his utility. In this model, δ_i does not appear, so we let $\delta_i = 1, \forall i$. The only source of uncertainty is the renewable energy $\{P_r(t)\}$. Fig. 1 shows the target demand profiles of $N = 4$ users in our simulation. The unit of energy is kWh.

$P_r(t)$ is uniformly distributed between 0 and $2 \cdot \bar{P}_r(t) > 0$, so that its mean is $E(P_r(t)) = \bar{P}_r(t)$. Also, $P_r(t)$'s are independent across t. The values of $(\bar{P}_r(t), \forall t)$ are (2, 3, 4, 5, 5, 6, 6, 7, 6, 5, 4, 3, 2, 2, 3, 4, 4, 4, 4, 3, 3, 2, 2, 2).

For each time period, assume that the cost functions are $c_d(P) = (P^2 + P)/2$, $c_o(P) = P/2$, and $c_b(P) = P^2/2 + 5P$.

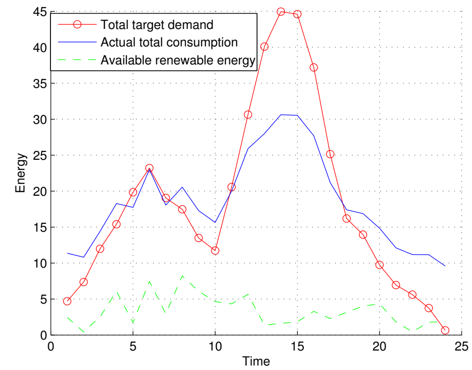


(a) Convergence of $P_d(6)$

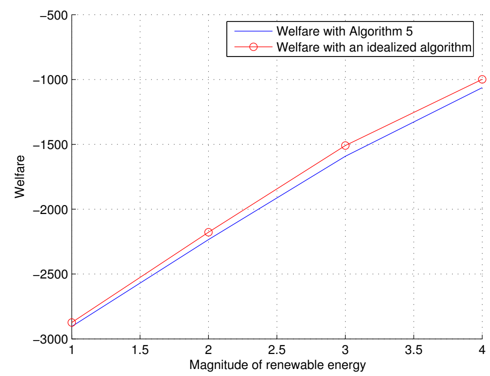


(b) Expected welfare in each time period

Fig. 2: Results of Algorithm 3



(a) Consumption



(b) Comparison with an idealized algorithm

Fig. 3: Results of Algorithm 5

A. *Without time correlation*

In the case without time correlation, we impose no constraint on the total consumption of each user i. This can model the scenario where users are willing to shed load instead of shifting load. We run Algorithm 3 for each time period. Fig. 2 (a) shows that the computed value of $P_d(6)$, the day-ahead energy of period 6, converges. For other t's, $P_d(t)$'s converges similarly. The expected social welfare under the converged $P_d(t)$ is given in Fig. 2 (b).

B. *With time correlation*

In this case, we impose the constraint that $\sum_t x_i(t) = \sum_i y_i(t)$. That is, user i can shift his demand from one time period to another, but his total consumption $\sum_t x_i(t)$ needs to be same as $\sum_i y_i(t)$.

We run Algorithm 5 with approximations (21) and (22), and obtain the consumptions of each user. Under one realization of $\{P_r(t)\}$, Fig. 3 (a) shows the total target load (of the 4 users), the total actual consumption, and the realization of $\{P_r(t)\}$. We observe that

- The consumptions tend to follow the trend of target demand profiles;
- but the users tend to consume more when there is more renewable energy, and at the same time even out their consumptions throughout the day.

We compare the result with an idealized algorithm which assumes that the realization of $\{P_r(t)\}$ is known ahead of time. The idealized algorithm is also allowed to choose $P_d(t)$'s according to the realization. With the $\{P_r(t)\}$ shown in Fig. 3, we find that the idealized algorithm achieves a total social welfare (summed over the 24 time periods) of -2873.9; whereas Algorithm 5 achieves a total social welfare of -2904.9. As expected, the idealized algorithm performs better than Algorithm 5. However, the difference is small. For further comparison, we increase the mean of renewable energy to twice, three times, and four times of the previous simulation (where we say that the magnitude of renewable energy is 1). The welfare achieved by Algorithm 5 and the idealized algorithm is shown in Fig. 3 (b). Note that the average renewable energy is 15.36% of the total target demand when the magnitude of renewable energy is 1, and is 61.45% when the magnitude is 4. Algorithm 5 achieves reasonable performance even when the magnitude is high.

VII. Conclusion

In this paper, we have studied multi-period energy procurement and demand responses in the presence of uncertain supply of renewable energy and uncertain demand. We have provided decentralized algorithms with two-way communications for the load-serving entity and the users, in order to achieve efficient use of the grid and maximize social welfare.

We have provided insight on the effect of clean, but variable renewable energy on the social welfare.

In the future, we would like to give more detailed considerations of the service requirements and properties of different appliances, and design more refined demand response algorithms accordingly. Also, we are interested in extend our model to the case with distributed renewable generations on the user side, and investigate how that changes the structure of energy procurement and demand response strategies.

REFERENCES

[1] US Department of Energy, "Benefits of Demand Response in Electricity Markets and Recommendations for Achieving Them," Report to the United States Congress, February 2006. Available online: http://eetd.lbl.gov

[2] M.H. Albadi, E.F. El-Saadany, "Demand Response in Electricity Markets: An Overview," Power Engineering Society General Meeting, 2007. IEEE , vol., no., pp.1-5, 24-28 June 2007.

[3] L. Chen, N. Li and S. H. Low, "Optimal Demand Response Based on Utility Maximization in Power Networks," submitted to PESGM 2011.

[4] P. Xu, P. Haves, M.A. Piette, and L. Zagreus, "Demand shifting with thermal mass in large commercial buildings: Field tests, simulation and audits,". 2005.

[5] J.E. Braun, "Load control using building thermal mass," Journal of solar energy engineering, 125:292, 2003.

[6] K. Clement-Nyns, E. Haesen, and J. Driesen, "The impact of charging plug-in hybrid electric vehicles on a residential distribution grid. Power Systems," IEEE Transactions on, 25(1):371–380, 2010.

[7] A. Mohsenian-Rad and A. Leon-Garcia, "Optimal residential load control with price prediction in real-time electricity pricing environments," IEEE Transactions on Smart Grid, 1(2):120–133, 2010.

[8] P.P.Varaiya; F.F. Wu, and J.W. Bialek, "Smart Operation of Smart Grid: Risk-Limiting Dispatch," Proceedings of the IEEE , vol.99, no.1, pp.40-57, Jan. 2011.

[9] M. He, S. Murugesan, and J. Zhang, "Multiple Timescale Dispatch and Scheduling for Stochastic Reliability in Smart Grids with Wind Generation Integration," to appear in Infocom'11 Mini-conference, Shanghai, China, Apr. 2011.

[10] S. Boyd and L. Vandenberghe, "*Convex Optimization*", Cambridge University Press, 2004.

[11] H. Robbins and S. Monro, "A Stochastic Approximation Method," The Annals of Mathematical Statistics, Vol. 22, No. 3, 400-407. Sep., 1951.

[12] J. Zhang, D. Zheng, M. Chiang, "The impact of stochastic noisy feedback on distributed network utility maximization," IEEE Transactions on Information Theory, vol. 54, no. 2, pp. 645-665, Feb. 2008.