A Unifying Market Power Measure for Deregulated Transmission-Constrained Electricity Markets

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***Abstract*—Market power assessment is a prime concern when designing a deregulated electricity market. In this paper, we propose a new functional market power measure, termed *transmission constrained network flow* (TCNF), that unifies three large classes of long-term transmission constrained market power indices in the literature: residual supply based, network flow based, and minimal generation based. Furthermore, it is suitable for demand-response and renewable integration and hence more amenable to identify market power in the future smart grid. The measure is defined abstractly, and so allows incorporation of power flow equations in multiple ways; this is illustrated this using: (a) a DC approximation, (b) a semidefinite relaxation, and (c) interior-point algorithms from Matpower. Finally, we provide extensive simulations on IEEE benchmark systems and highlight the complex interaction of engineering constraints with market power assessment and compare all the different approaches to assess market power.**

***Index Terms*—Market power, electricity markets.**

I. Introduction

Beginning in 1990 with Chile, electricity markets in many regions have moved from being a vertically integrated regulated monopoly to a deregulated market structure that encourages innovation and competition in technology. The California energy crisis in 2000-01, however, highlights how strategic interaction can shadow the benefits of competition in a deregulated market. It is estimated that about $5.55 billion was paid in excess of costs in the deregulated market in California between 1998 and 2001 alone [1]. To avoid such over-payments, monitoring and mitigating market power is essential. It is expected to become even more critical as new smart grid technologies such as intermittent renewable generation, energy storage, and demand-response programs start picking up presenting more opportunities to exploit.

The Department of Justice defines market power as the ability of a firm to profitably alter prices away from competitive levels [2], [3]. In other words, market power is a form of market "dominance", where a player can increase its profitability by behaving independently of competitors and consumers. Market power in generic markets has been extensively studied using microeconomics, e.g., in [4]. The theory, however, does not apply directly to *electricity markets* due to various reasons, such as: (a) Unlike in most commodity markets, electricity cannot be stored cheaply; therefore generators have significant short-run capacity constraints. (b) Electricity demand is typically inelastic because of limited price-responsiveness of consumers. (c) Trade agreement between a supplier and a consumer is not enough to guarantee feasible power delivery over a transmission grid since power transfer respects physical laws as well as market outcomes. Economics or engineering alone cannot handle such issues adequately. In electricity market, such dominance can be global, e.g., by a power supplier with a large enough generation capacity, or local, e.g., by a power supplier in a region which has limited ability to import less expensive electricity due to transmission constraints [5].

Classically, the literature on market power is fractured. Recently, however, a principled design has begun to emerge, e.g., see [3] for a survey. The analysis can be divided into two distinct categories: (a) long-term analysis [6]–[12] and (b) short term analysis [12]–[15]. The long-term analysis of market power is an *ex-ante* approach to study potential market power in capacity markets and is useful to evaluate mergers, plan transmission capacity expansion, and identify "must-run" generators. Short-term analysis is an *ex-post* approach to identify misconduct of generation firms in the spot market and detect supply withholding. In this work, our focus is on long-term market power.

The main contribution in this paper is to introduce a *functional* long-term market power measure that, for the first time, incorporates a detailed AC model of the underlying power system. [1] The new measure, termed "transmission constrained network flow" unifies the three broad classes of long-term measures in the literature: "network flow based" [11], [17], "residual supply based" [12], [18], and "minimal generation based" [19], [20]. We introduce each of these classes in detail in Section II. Calculating the new measure in Section III requires us to solve a nonconvex optimization program resulting from the nature of the AC power flow equations. We deal with this nonconvexity in three ways: (a) use DC approximation [21], [22] and solve a linear program (LP) , (b) use interior-point based methods implemented in Matpower [23], (c) use recent advances in semidefinite programming (SDP) based relaxations [24] to AC power flow equations [25]–[28]. In Section IV, we provide extensive simulations on IEEE benchmark systems [29] and illustrate the importance of modeling engineering constraints in identifying market power. We compare the different computational approaches in Section V and extend the index to the case where firms can own generators at multiple locations in Sec VI.

II. Market power measures

Early work on long term market power analysis, emerging from microeconomic theory, suggested measures that ignore transmission constraints. Bushnell et al. introduced the *pivotal supplier index* (PSI) as a binary indicator examining whether

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[1]A preliminary version of this work has appeared in [16].

the capacity of a generator is larger than the supply surplus, i.e., the difference between the total supply and the total demand [6]. Later, Sheffrin et al. refined PSI by measuring market power on a continuous scale, and proposed the *residual supply index* (RSI) in [7]. This index is used by the California ISO to assure price competitiveness [8]. The electric reliability council of Texas (ERCOT) uses a different measure, the *element competitiveness index* (ECI) [9], which is based on the *Herfindahl-Hirschmann index* (HHI) [10].

These indices measure market power purely in terms of generation capacity and fail to identify the impact of transmission capacities of lines, which are crucial to study market power in electricity markets since congested transmission lines can create geographically isolated pockets where a supplier can exploit the limited ability to import inexpensive generation from other areas. Recently, many indices have been introduced to include the effect of transmission constraints; we categorize them as: "residual supply based", "network flow based", and "minimal generation based".

A. Residual supply based measures

As in [12], [18], residual supply based measures propose to quantify the maximum total load that the transmission-constrained electricity market can meet if generator of interest, s, is excluded. In particular, the *transmission-constrained residual supply index* (TCRSI) for generator s is defined as:

$$\begin{aligned} \text{TCRSI}_s = \underset{q,t}{\textbf{maximize}} \quad & t \\ \textbf{subject to} \quad & \mathbf{1}^\dagger q = \mathbf{1}^\dagger(\bar{d}t), \\ & -b \le H_q q - H_d(\bar{d}t) \le b, \\ & q_s = 0, \quad 0 \le q_i \le \bar{q}_i, i \neq s. \end{aligned} \tag{1}$$

where q is the supply vector, t is the demand scaling parameter, H_q is the generation shift factor matrix, H_d is load shift factor matrix, b is the transmission line capacity vector, $\bar{q}_i$ is the capacity of generator i, $\bar{d}_j$ is the demand of load j, $\mathbf{1}$ is a unit vector, and $\dagger$ denotes transposition. If $\text{TCRSI}_s < 1$, then generator s may gain market power. Consider the network in Figure 1. For G_1, TCRSI is $3.2/7$, the fraction of demand that can be met with available supply.

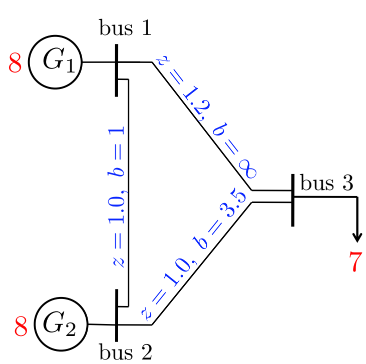


Fig. 1: A small network to illustrate market power indices. All quantities are measured in per units (p.u.). z denotes impedance and b denotes line capacity.

B. Network flow based measures

Network flow based measures are exemplified by [11], [17], which model market power in the presence of transmission constraints in terms of the *maximal network flow* (MNF) achievable without the generator of interest. Conceptually, these measures are similar to TCRSI, but they do not use power flow equations to model the underlying power systems. A key result in [11], [17] is that market power is supermodular, i.e., there is always an incentive for generators to collude. This conclusion, however, does not hold if the power flow respects impedance and follows Kirchoff's laws; the network in Figure 1 is a counterexample. See Section VI for an example in IEEE test systems.

C. Minimal generation based measures

The above two definitions of market power focus on the fraction of unmet demand when generator at bus s is not in service. An alternate approach is to calculate the minimum generation required from generator s to meet the total target demand. In particular, minimal generation based measures typically identify "must run generators", e.g., [19], [20] are exemplified by the *transmission-constrained minimal generator index* (TCMGI):

$$\begin{aligned} \text{TCMGI}_s = \underset{q}{\textbf{minimize}} \quad & q_s \\ \textbf{subject to} \quad & \mathbf{1}^\dagger q = \mathbf{1}^\dagger \bar{d}, \\ & -b \le H_q q - H_d \bar{d} \le b, \\ & 0 \le q_i \le \bar{q}_i. \end{aligned} \tag{2}$$

Note that in (1), we have $q_s = 0$ and the total load is scaled by a variable factor t. In (2), however, the output of generator s is a variable and the total demand is a constant. If $\text{TCMGI}_s > 0$, then generator s may gain market power. In general, TCMGI_s does *not* equal the unmet demand in the network when generator at bus s is not operational. For example, consider the network in Figure 1. It can be checked that $\text{TCMGI}_1 = 4.2pu$ while the shortfall is actually $3.8pu$ when the same generator is not in service. TCRSI_s and TCMGI_s are indeed related; we explore this in the next section.

III. Functional Measure of Market Power

Prior work on long term market power measures in Section II suggests that while a wide variety of measures exist, the literature is quite fractured and lacks a unified theory that incorporates economic and engineering constraints. Here we propose a *functional* market power measure rather than a market power *index* that represents a step toward such a unifying measure.

To motivate the measure, consider the following informal definition:

$$\begin{aligned} \text{TCNF}_s(\rho) = \textbf{maximize} \quad & \text{total demand met} \\ \textbf{subject to} \quad & \text{supply from generator } s \le \rho, \\ & \text{other network constraints.} \end{aligned}$$

The functional TCNF_s maps every scalar $\rho \in [0, \bar{q}_i]$ into the maximum demand that can be satisfied when the (real) power output of generator s is no more than ρ. $\text{TCNF}_s(\rho)$ can also be interpreted as a measure of the minimum amount of load that has to be shed (or dispatched, through demand-side management[2]), provided that the supply of generator s is up to

[2]When there is a deficit in electricity supply, the system operator may call upon consumers to adjust their demand so as to match the supply — an approach which is usually referred to as demand response.

ρ. At different levels of ρ, it measures the relative importance of each generator to meet additional demand, abiding by the network constraints.

In the rest of this section, we show how a detailed power flow model can be included to arrive at a unifying market power measure that is applicable to the evolving smart grid. Next, we formally define $\text{TCNF}_s(\rho)$ with the engineering constraints.

A. Definition

We begin with some notation. Let $\mathbf{i} = \sqrt{-1}$ and for any complex matrix or number z, let z^{H} be the complex conjugate transpose of z. Consider a network on n nodes (buses) labeled $1, 2, \ldots, n$. Let p_k^G and q_k^G be the real and reactive power generations at node k. Also let p_k^D and q_k^D be the real and reactive power demands that are met at node k. We denote $s_{kj} := p_{kj} + \mathbf{i}q_{kj}$ as the apparent power flowing from bus k to bus j, where p_{kj} and q_{kj} are the real and reactive power flows, respectively. Thus, power balance equation at each node k becomes

$$(p_k^G - p_k^D) + \mathbf{i}(q_k^G - q_k^D) = \sum_{j:j\sim k} s_{kj}, \tag{3}$$

where $j \sim k$ denotes that buses k and j are connected in the power network. The power generations are assumed to satisfy

$$0 \leq p_k^G \leq \overline{p}_k^G, \qquad -\beta_k \; p_k^G \leq q_k^G \leq \beta_k \; p_k^G, \tag{4}$$

where $\beta_k > 0$ is a known constant that depends on the technology, i.e., each generator is assumed to vary its reactive power output within a certain power factor of the real power generation [30]. The total load to be supported at bus k has a target real demand $\overline{p}_k^D$ and a target power factor α_k. The target power factor depends on the type of load at bus k. Thus, the supported demand $p_k^D + \mathbf{i}q_k^D$ satisfies

$$0 \leq p_k^D \leq \overline{p}_k^D, \qquad q_k^D = \tan(\cos^{-1}\alpha_k)\; p_k^D. \tag{5}$$

Power factors typically range from 0.95 to 0.98 lagging. The real power flowing from bus k to bus j is p_{kj} and is bounded by the thermal and stability limits of the transmission lines as

$$|p_{kj}| \leq f_{kj}, \tag{6}$$

where f_{kj} is the known capacity of the line between buses k and j.[3] Let the voltage at bus k be V_k, and the admittance of the line between buses k and j be y_{kj}. The current flowing from bus k to bus j is $y_{kj}(V_k - V_j)$ and we have

$$s_{kj} = V_k \left[y_{kj}(V_k - V_j)\right]^{\text{H}}. \tag{7}$$

To maintain power quality and the system stability, the voltage magnitude $|V_k|$ at bus k is required to be bounded as follows:

$$\underline{W}_k \leq |V_k|^2 \leq \overline{W}_k, \tag{8}$$

where $\underline{W}_k$ and $\overline{W}_k$ are known constants.

Using the notations introduced above, we are now ready to formally introduce a measure the market power of a generator at node s as follows:

$$\begin{aligned} \text{TCNF}_s(\rho) = \textbf{maximize} \quad & \sum_k p_k^D \\ \textbf{subject to} \quad & p_s^G \leq \rho, \\ & (3), (4), (5), (6), (7), (8), \\ \textbf{over} \quad & p_k^G, q_k^G, p_k^D, q_k^D, k = 1, \ldots, n, \\ & s_{kj}, \quad k \sim j. \end{aligned} \tag{9}$$

We refer to this measure as the *transmission-constrained network flow*. The constraints in (3)-(8) impose the impact of the network topology, the underlying circuits, and the transmission line capacities. These constraints make our analysis different from a traditional economic approach to market power. Note that, $\text{TCNF}_s(\rho)$ is a *functional* measure, i.e., it evaluates market power for every given value of parameter ρ.

In Section III-C, we describe how this measure in (9) unifies the three general classes of long-term market power measures discussed in Section II. Next, we describe the solution approaches to the optimization program to calculate TCNF_s.

B. Relaxations and approximations

Perhaps the first observation one makes about the definition of TCNF_s is that it requires solving optimization problems that are NP-hard. This is because the relation in (7) is a quadratic equality and hence the feasible set is, in general, non-convex. This makes it difficult to compute (9) to quantify market power. There are three general approaches to compute the measure: (i) linearizing the quadratic constraint around a set-point and use DC approximation (ii) using heuristic iterative nonconvex optimization techniques, (iii) relaxing the non-convex quadratic equality constraint to a convex semidefinite constraint and use conic program solvers.

Nonconvexity of power-flow equations have played a significant role in optimization problems over power networks [31]. Traditionally, the engineering problems and market computations have differed in the approaches taken to deal with this nonconvexity. While market outcomes have relied on the DC approximation [6], [7], [9], [11], [12], [16], engineering problems such as real-time economic dispatch have applied heuristics or iterative techniques to reach an implementable operating point [23], [32]. The conic relaxation approach, however, is a recent development and is finding applications in both the engineering and market considerations, e.g., see [25], [26], [28] for its use in optimal power flow and see [33], [34] for its use in electricity markets. Next, we present all three computational approaches; we compare them in Section V.

1) The DC approximation approach: The most popular approximation for power flow equations is linearization, e.g., see [21], [22]. This approach makes the following assumptions:

- Voltage magnitude $|V_k|$ at each node k is assumed to be at its nominal value, where $V_k = |V_k|\exp(\mathbf{i}\theta_k)$. Thus $|V_k| = 1pu$.
- Transmission lines are assumed to be loss-less, i.e., $y_{kj} = \mathbf{i}b_{kj}$ is purely imaginary for all pairs $k \sim j$.
- For any pair of buses $k \sim j$, the voltage phase angle differences $\theta_k - \theta_j$ are assumed to be small, i.e., $\sin(\theta_k - \theta_j) \approx \theta_k - \theta_j$ and $\cos(\theta_k - \theta_j) \approx 1$.

Using this approximation, for any pair $k \sim j$, we have

$$s_{kj} = p_{kj} = b_{kj}(\theta_k - \theta_j).$$

[3] Another common practice is to limit the apparent power flow, i.e., $\sqrt{p_{kj}^2 + q_{kj}^2} \leq f_{kj}$. In this work, however, we omit such constraints.

It can be checked that there is no reactive power that flows in this model and hence ignoring the reactive power demand constraint in (5), this definition of TCNF$_s$ coincides with the one studied in [16] and can be solved as an LP. Henceforth, we refer to this computation as the *DC case*, denoted by TCNF$_s^{DC}(\rho)$.

2) Non-linear optimization technique: Many iterative techniques have been used to solve optimization problems in power systems, specifically the optimal power flow problem; see [31], [32] for surveys. Some notable examples are quadratic programming, variations of gradient methods, Newton-based techniques, sequential quadratic programming, and interior-point based methods. The problem is NP-hard, these iterative algorithms are not guaranteed to converge to the global optimal solution, though some of them provably converge to local minima in polynomial time. For many test cases, these approaches have been known to converge to "good" operating points. In this work, we use the primal-dual interior-point solver in Matpower [23]. When this converges, we refer to it as TCNF$_s^{NL}$ and call this computation as the *NL case*. Though it is hard to comment on the optimality of the point obtained through this heuristic, the use of Matpower solver provides insights as we explore the simulations on the IEEE benchmark systems.

3) The SDP relaxation approach: Recently, a conic relaxation has been proposed to deal with the nonconvexity of power-flow equation in (7), e.g., see [25]–[28]. In particular, consider the $n \times n$ positive semidefinite matrix $W = VV^{\mathrm{H}}$ that has rank one (denoted as $W \succeq 0, \text{rank } W = 1$). For each pair of buses $k \sim j$, we express s_{kj} as a linear matrix relation in W as follows. Define an $n \times n$ matrix M^{kj}, where

$$[M^{kj}]_{kk} = y_{kj}^{\mathrm{H}}, \quad [M^{kj}]_{jk} = -y_{kj}^{\mathrm{H}},$$

and rest of the entries of M^{kj} are zero. In terms of M^{kj}, the equality in (7) can be written as

$$s_{kj} = \mathrm{tr}(M^{kj}W) = p_{kj} + \mathbf{i}q_{kj}.$$

Accordingly, the optimization problem to calculate TCNF becomes a rank-constrained SDP [24] in terms of matrix W. It still remains nonconvex due to the rank constraint. Next, we relax the rank constraint to obtain TCNF$_s^{AC}(\rho)$ and refer to this computation as the *AC case*.

C. Properties of TCNF$_s$

In Section III-B, we introduced the functional measure TCNF$_s(\rho)$ and its computational versions TCNF$_s^{DC}(\rho)$, TCNF$_s^{AC}(\rho)$ and TCNF$_s^{NL}(\rho)$ to assess market power. Now, we explore their salient features.

TCNF$_s^{DC}(\rho)$ generalizes network flow based and residual supply based measures. When $\rho = 0$, it indicates the maximal network flow satisfying the DC power flow constraints when generator s withholds generation. TCNF$_s^{AC}(0)$ and TCNF$_s^{NL}(0)$ measure the same with AC power flow models.

To relate TCNF$_s$ to the minimum generation based measure, consider the *transmission-constrained minimal generation* TCMG$_s(D)$ for generator s to be defined as follows:

$$\begin{aligned} \mathrm{TCMG}_s(D) = \textbf{minimize} \quad & p_s^G, \\ \textbf{subject to} \quad & \sum_k p_k^D = D, \\ & (3), (4), (5), (6), (7), (8), \\ \textbf{over} \quad & p_k^G, q_k^G, p_k^D, q_k^D, k = 1, \ldots, n, \\ & s_{kj}, \ k \sim j. \end{aligned}$$

This generalizes the minimum generation based measures in [11], [17] to a functional form and uses AC power flow to model the physical power system. It is easy to extend the definition of TCMG$_s(\cdot)$ to the following computable versions: TCMG$_s^{DC}(\cdot)$ with the DC-approximation and TCMG$_s^{AC}(\cdot)$ with the SDP-based relaxation. First, we explore the relationship of the functions TCNF$_s(\cdot)$ and TCMG$_s(\cdot)$ for the DC and the AC cases; proof is omitted for brevity.

Theorem 1. *For each generator s:*

1) *TCNF$_s^{DC}(\cdot)$ is a continuous, concave, piecewise linear and non-decreasing function; TCMG$_s^{DC}(\cdot)$ is a continuous, convex, piecewise linear and non-decreasing function. Moreover, TCNF$_s^{DC}(\cdot)$ and TCMG$_s^{DC}(\cdot)$ are inverses of each other, i.e., for any $0 \leq D \leq$ TCNF$_s^{DC}(\infty)$,*

$$\mathit{TCNF}_s^{DC}\left[\mathit{TCMG}_s^{DC}(D)\right] = D.$$

2) *TCNF$_s^{AC}(\cdot)$ is a continuous, concave, and non-decreasing function; TCMG$_s^{AC}(\cdot)$ is a continuous, convex, and non-decreasing function. Moreover, TCNF$_s^{AC}(\cdot)$ and TCMG$_s^{AC}(\cdot)$ are inverses of each other, i.e., for any $0 \leq D \leq$ TCNF$_s^{AC}(\infty)$,*

$$\mathit{TCNF}_s^{AC}\left[\mathit{TCMG}_s^{AC}(D)\right] = D.$$

We make a few observations about the result. Note that the functions TCNF$_s^{DC}(\cdot)$ satisfies all properties of TCNF$_s^{AC}(\cdot)$; in addition, it is also piecewise linear, because the optimization problem to compute TCNF$_s^{DC}$ is a linear-parametric LP. This property does not generalize to linear-parametric SDPs; see [24] for a counterexample.

The inverse relationship between TCNF$_s^{DC}(\cdot)$ and TCMG$_s^{DC}(\cdot)$ holds for all $0 \leq D \leq$ TCNF$_s^{DC}(\infty)$. Here, TCNF$_s^{DC}(\infty)$ is the total demand in the network that can be met when the power generated by generator s is not constrained (it, however, satisfies the generation capacity constraint $0 \leq p_s^G \leq \overline{p}_s^G$ in (4)). Beyond that, the network cannot satisfy the target demand and hence TCMG$_s^{DC}(D)$ only exists for $0 \leq D \leq$ TCNF$_s^{DC}(\infty)$. Similar result holds for the AC case. Unlike the DC and AC approximations, when TCNF$_s(\cdot)$ is instantiated with the true AC power flow equations (i.e., not the SDP relaxation), then it may not be concave since the feasible set of the corresponding optimization problem is not convex. The function TCNF$_s(\cdot)$ may not be monotonically increasing in the interval $[0, \mathrm{TCMG}_s(\mathrm{TCNF}_s(\infty))]$ in this case, and thus not invertible. The NL case is similar.

Next, we illustrate the result of Theorem 1 through an example. Consider the network shown in Figure 1. TCNF$_s^{DC}(\cdot)$ is plotted for generators at buses 1 and 2 in Figure 2. Functions TCNF$_1^{DC}(\cdot)$ and TCNF$_2^{DC}(\cdot)$ are continuous, convex, piecewise linear and non-decreasing. As noted earlier, TCNF$_s^{DC}(0)$ equals the TCRSI for generator s. Also, TCMGI for generator

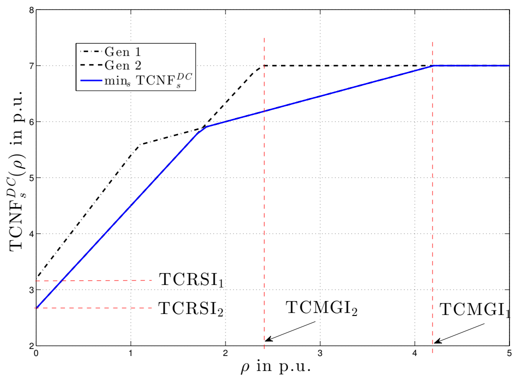


Fig. 2: TCNF$_s^{DC}(\cdot)$ of generators in the 3-bus network shown in Figure 1. Quantities are measured in per units (p.u.).

s is given by $\min\{\rho \geq 0 \mid \text{TCNF}_s(\rho) = \text{TCNF}_s(\infty)\}$. TCRSI and TCMGI for each generator are indicated in the figure.

Lower the value of TCNF$_s^{DC}(\cdot)$, higher the market power of generator s. Thus we plot $\min_s$ TCNF$_s^{DC}$ by considering the lower envelope of TCNF$_1^{DC}(\cdot)$ and TCNF$_2^{DC}(\cdot)$ to indicate the market power of the dominant generator for each $\rho \geq 0$. In this example, the generator with maximum market power changes as ρ changes. This suggests that market power assessment is complex and cannot be sufficiently captured through a single index. Note that ρ can also indicate the available capacity of an intermittent renewable source of energy like wind or solar. Our functional measure thus provides a meaningful way to capture the complexity of a future smart grid.

IV. Case Studies

In this section, we use our proposed unifying measure to assess market power of generators in various IEEE test systems [29]. In particular, we show how market power can be affected by different factors such as the variation of target demand due to distributed renewable generation, changes in dispatchable load in presence of demand-response programs, and changes in load power-factors. We also compare the results obtained from TCNF$_s^{DC}$, TCNF$_s^{AC}$ and TCNF$_s^{NL}$ and point out the important role of nonconvexity of power flow equations in assessing market power. This underlines the significance of engineering constraints on market outcomes in electricity markets.

In our simulations, we consider the IEEE 6-bus and 39-bus test systems. In each case, we look at a variety of scalings of the target demands in the test system to understand the impact of demand fluctuation and distributed renewable generation. Specifically, target demands are scaled uniformly by a scalar $t \geq 0$. We assume that for all generators, the minimum level of generation is zero, i.e., $p_k^G \geq 0$. Most systems have a reactive generation capability defined by $\underline{q}_k^G \leq q_k^G \leq \overline{q}_k^G$. We modify this box constraint on q_k^G to $-\beta_k p_k^G \leq q_k^G \leq \beta_k p_k^G$ as in (4), where β_k is chosen accordingly for each case study. To compute TCNF$_s^{DC}(\cdot)$ and TCNF$_s^{AC}(\cdot)$, we use the convex programming package CVX [35] in MATLAB. The SDP solver is SeDuMi [36]. Finally, TCNF$_s^{NL}$ is computed using the primal-dual interior-point method in Matpower [23].

A. *IEEE 6-bus Test System*

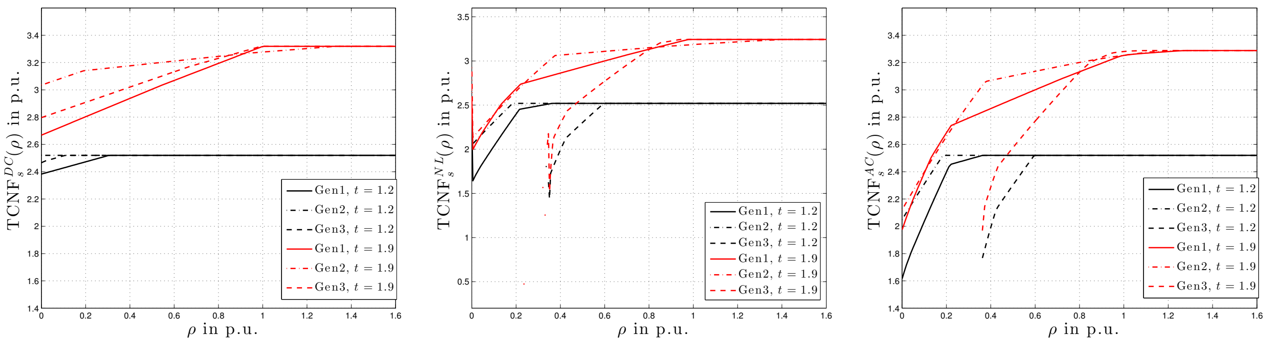
The IEEE 6-bus test system has three generators at nodes 1, 2 and 3. For all generators, we assume that $\beta_k = 0.6$ and for all loads we assume that the power-factors are $\alpha_k = 0.98$ lagging. In Figure 3, we plot TCNF$_s^{DC}(\rho)$, TCNF$_s^{NL}(\rho)$, and TCNF$_s^{AC}(\rho)$ for demand scalings of $t = 1.2$ and $t = 1.9$. Note that, there is a remarkable difference between the AC and the DC cases, while the results from the NL case is similar to that of the AC model. Therefore, we can conclude that in this case study, the SDP relaxation finds a solution very close to or inside the feasible region of the non-convex feasible set of the optimization problem in (9).

In Theorem 1, the TCNF functions for the DC and AC cases in Figure 3 are increasing and concave for all generators. This property does not generalize for the NL case. Note that, for generator 3, the optimization problem for calculating TCNF$_s^{AC}$ remains infeasible for $\rho \leq 0.35pu$. This indicates that generator 3 is needed to supply at least $0.35pu$ in order to maintain system stability. It is interesting to note that if the SDP relaxation is infeasible, so is the non-linear optimization problem in (9) and hence the interior-point method does not converge to a feasible point for $\rho \leq 0.35pu$. We can also see that TCNF$_s^{NL}$ and TCNF$_s^{AC}$ are quite similar except for generators 1 and 2 at $\rho = 0$, where TCNF$_s^{NL}$ is greater than TCNF$_s^{AC}$. For such a non-convex optimization problem, determining feasibility is NP-hard and hence it is hard to comment whether the problem in (9) is infeasible at $\rho = 0$. The SDP relaxation TCNF$_s^{AC}$, however, is feasible. Moreover, it is continuous at $\rho = 0$ as expected from Theorem 1.

The results in Figure 3 can be further interpreted as follows. For the AC case at $t = 1.9$, consider generators 1 and 3 at a total demand level (y-axis) of $3pu$. At this demand level (which is lower than the total target demand level), TCNF$_3^{AC}$ has a larger slope than TCNF$_1^{AC}$. Therefore, in order to satisfy an extra unit of demand at $3pu$, generator 3 has to supply less additional power and hence it is more valuable to the system operator. This means that generator 3 has more market power in an *incremental market*.

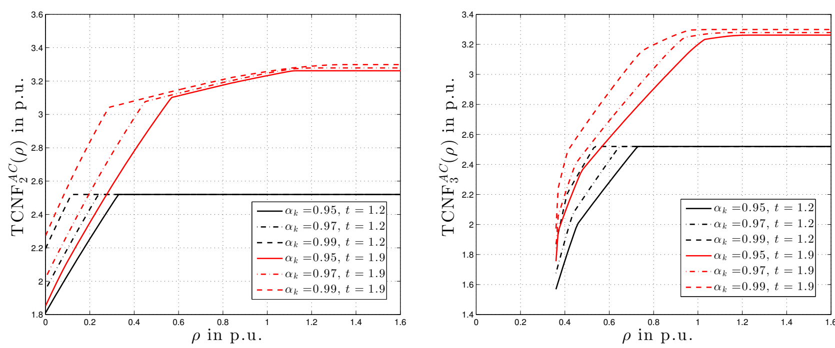
Another key observation is about the importance of each generator at various demand levels, in presence of *dispatchable load*. In this regard we see that at the same demand level, TCNF$_s^{DC}$ in Figure 3(a) and TCNF$_s^{AC}$ in Figure 3(c) give conflicting conclusions, while TCNF$_s^{NL}$ in Figure 3(b) agrees with TCNF$_s^{AC}$, indicating that the relaxation approach of AC power flow model is efficient in quantifying market power in the IEEE 6-bus system. This is yet another evidence that the market outcome is heavily dependent on the underlying power engineering model.

Finally, in order to illustrate the importance of reactive power flows, consider TCNF$_s^{AC}(\rho)$ in Figures 4(a) for generator 2 and in Figure 4(b) for generator 3, respectively. The plots have been generated with $t = 1.2$ and $t = 1.9$ and the load power factors have been varied from 0.95 to 0.99 lagging uniformly for all buses in each case. TCNF$_2^{AC}(\rho)$ and TCNF$_3^{AC}(\rho)$ show considerable variations with changes in load power factors and thus reactive power flow has a significant effect on market power and must be taken into consideration for an efficient long-term planning. For example, as load power factor decreases, generators 1 and 3 are needed to supply more power in order to meet the same level of demand, placing these generators in better positions to gain market power. Another interesting observation is that although changing the load power factor can significantly change the slope of the TCNF$_s^{AC}(\rho)$ function at different points, it does



(a) TCNF with DC power flow (b) TCNF with Matpower (c) TCNF with AC power flow

Fig. 3: TCNF calculation based on different approaches for various generators in the IEEE 6-bus system.



(a) TCNF^{AC} of generator 2 (b) TCNF^{AC} of generator 3

Fig. 4: $\mathrm{TCNF}^{AC}(\rho)$ for generators 2 and 3 plotted for load power factors from 0.95 to 0.99 lagging in the IEEE 6-bus system.

not have direct impact on the cut off rate of $\mathrm{TCNF}_3^{AC}(\rho)$, i.e., the choice of ρ for which the optimization problem in (9) for $s = 3$ becomes infeasible. Similar results can be observed for $\mathrm{TCNF}_s^{NL}(\rho)$ (not shown here).

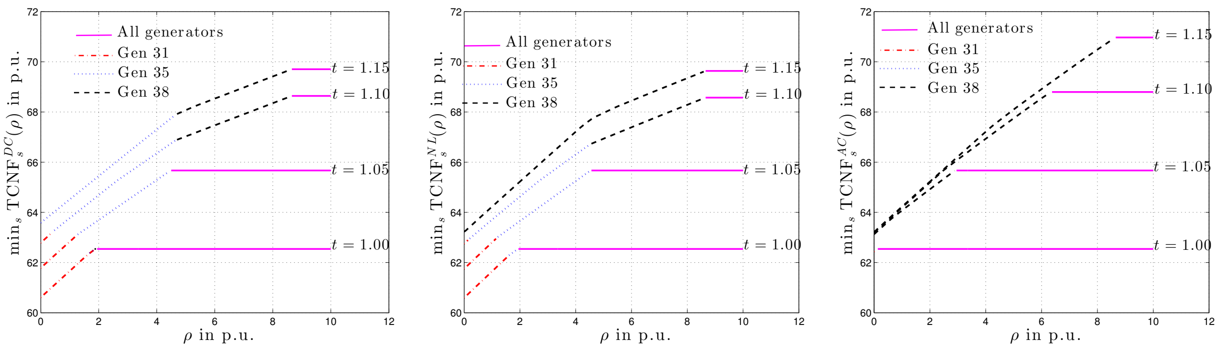
B. IEEE 39-bus Test System

We now assess our proposed approach for market power analysis in a larger IEEE test system with 39 buses. Here, we are particularly interested in examining the benefit of adding dispatchable load at different buses as far as mitigating market power is concerned. First, we note that at each bus s, the value of parameter ρ in function $\mathrm{TCNF}_s(\rho)$ can be interpreted as the amount of curtailable load that is available for dispatch at bus s, in case of losing the generator at bus s. Of course, the higher the amount of dispatchable load at a bus, the better the grid operator can handle the loss of a generator at that bus, preventing such generator from gaining market power. However, the effectiveness of the same amount of dispatchable load in mitigating market power may not be the same at different buses. In other words, dispatchable load can be more (or less) valuable at certaint locations. For example, consider the simulation results in Figure 5. Here, we are considering the TCNF_s for generators at buses 31, 35 and 38. For the purpose of our analysis, we plot the lower envelope of TCNF_s only, namely $\min_{s\in\{31,35,38\}} \mathrm{TCNF}_s(\cdot)$ for demand levels ranging from $t = 1.0$ to $t = 1.15$. For the case where $t = 1.15$, increasing the dispatchable load capacity is most beneficial when it is done at bus 38 because the generator at bus 38 has the highest potential to gain market power in this case. As another example, for the case where $t = 1.05$, if there is $1pu$ of dispatchable load capacity already in place in all generator buses, then increasing the dispatchable load capacity is most beneficial at bus 31, but if there is $3pu$ of dispatchable load capacity already in place in all generator buses, then increasing the dispatchable load capacity at bus 35 is most beneficial. The impact of adding renewable generation resources at different buses can be examined similarly, as they too directly impact the operating point ρ and in turn, market power.

Besides analyzing the impact of dispatchable load and renewable generation resources, we make the following observations based on the results in Figure 5: (a) In the DC approximation case, depending on the value of ρ, different generators may gain the maximum market power. However, in the AC case, it is only generator 38 that always maintains the maximum market power for all values of ρ. (b) The DC and the NL cases are more similar to each other than the corresponding AC case. (c) For demand scaling of $t = 1.15$, the DC and NL cases indicate that the total demand that can be met is lower than the total target demand. In the AC case, however, the total target demand of about $71.1pu$ can be satisfied.

V. Comparison of Computational Approaches

Now that we have presented our simulation results, we discuss the relative pros and cons of the three computational approaches to evaluate market power, namely, $\mathrm{TCNF}_s^{DC}(\cdot)$, $\mathrm{TCNF}_s^{NL}(\cdot)$ and $\mathrm{TCNF}_s^{AC}(\cdot)$.



(a) TCNF with DC power flow

(b) TCNF with Matpower

(c) TCNF with AC power flow

Fig. 5: The lower envelope of TCNF, i.e., $\min_s \text{TCNF}_s$ for selected generators in the IEEE 39-bus system.

DC Approximation Case: This approach uses an approximation of the underlying system and formulates the optimization as an LP that is fast and scalable with the size of the network. Since $\text{TCNF}_s^{DC}(\cdot)$ is continuous and piecewise linear, we can characterize the slopes of the linear segments of $\text{TCNF}_s^{DC}(\cdot)$ using Lagrangian duality [24]; furthermore, we can use these slopes to provide an efficient way to compute the function. Specifically, for generator s, let μ be the Lagrange multiplier for the constraint $p_s^G \leq \rho$. For any function $f(z)$ in variable z, define $(df(z)/dz)^+$ as its right-hand derivative . We can relate the slopes of the linear segments of the functions $\text{TCNF}_s^{DC}(\rho)$ as follows:

$$\left(\frac{d}{d\rho} \text{TCNF}_s^{DC}(\rho) \right)^+ = \mu_*, \tag{10}$$

where μ_* is the Lagrange multiplier at the optimum. Recall that $\text{TCNF}_s(\rho)$ is piecewise linear and is non-differentiable at the end-points of each line segment, but the right-hand derivative in (10) is well-defined. Using (10), a recursive algorithm can be developed to compute $\text{TCNF}_s^{DC}(\rho)$ for ρ in any interval $[a, b]$; see [16] for details.

Using Matpower: Matpower is a MATLAB toolbox that implements a primal-dual interior-point algorithm to solve the power flow equations [23]. Interior-point methods were popularized by Karmarkar for LPs [37] and Nesterov et al. for SDPs [38]. For LPs and SDPs, it is proved that interior point methods converge to a *global* optimal solution in polynomial time. For nonlinear nonconvex problems, they rather provide a heuristic approach to obtain a *local* optimal solution. Matpower has been known to perform well for economic dispatch problems over various IEEE test systems. As evidenced by our simulations, the NL case often shows similarity to the DC and the AC cases and provides a yard stick to measure the performance of our proposed DC approximation and the AC relaxation approaches. However, we reiterate that computing TCNF in (9) is NP-hard and thus it is hard to comment on the optimality of the solution obtained using Matpower.

AC Relaxation Approach: The DC approximation completely ignores the reactive power flows; our studies on IEEE benchmark systems, however, indicate that reactive power flows play an important role in characterizing market power potential. To tackle such limitations, we use the SDP relaxation approach with an AC power flow model. When the relaxation is exact, it indeed provides a global optimal solution as opposed to the heuristic NL case. The sufficient conditions for exact relaxation, however, are specific to particular network topologies and constraint patterns [26], [27]. When line-flow constraints are active, the relaxation is often inexact, as in [39] and the optimization yields a non rank-1 optimal W_*. We encounter similar results in our simulations. To better understand the accuracy of our simulations, we report the statistics of the quantity $\eta := \lambda_2(W_*)/\lambda_1(W_*)$ for the IEEE benchmark systems in Table I, where $\lambda_1(W_*), \lambda_2(W_*)$ are the first and second eigenvalues of the positive semidefinite matrix W_*, respectively. A lower value for this ratio indicates a smaller optimality gap and hence more accurate results. We see that η is typically very small in our simulations; although it may not always be negligible. Such optimality gaps may not be accurate to find operating points in economic dispatch, but as far as long-term market power analysis is concerned, the results are accurate enough to provide valuable insights to the system planner that is often not obvious using the DC power flow model. The SDP relaxation approach is known to scale poorly with the size of the network. Recent results in [28], [40], however, suggest that the sparsity of the power network can be exploited suitably to obtain scalable conic relaxations that are fast enough to compute on large networks.

Test Case	# of Scenarios	Mean η	Max η
6-bus	556	0.0051	0.0176
9-bus	600	0.0058	0.0249
39-bus	900	0.0096	0.0173

TABLE I: Statistics of η for IEEE benchmark systems.

VI. Firm behavior

Our focus so far has been on identifying market power of a single generator. However, our analysis can easily be extended to the case where a single firm owns multiple generators at different locations. Let $\mathcal{S}$ denote the set of locations (buses) where the firm has a generator. The TCNF index of the firm can be defined using the optimization problem (9) with a modified constraint that the *total* supply of the firm's generators does not exceed ρ, i.e., $\sum_{s \in \mathcal{S}} p_s^G \leq \rho$. Similarly, the TCMG index of a firm can be defined as the minimum *total* supply needed from the generators of this firm in order to meet a certain demand level D. This index can be calculated by modifying the objective function to $\sum_{s \in \mathcal{S}} p_s^G$ in the definition of TCMG_s.

Note that, if an "adversarial" firm acts strategically to degrade the performance of the grid, then the behavior of each individual generator (of the firm) might be potentially different if it acted as a separate entity. A game theoretic analysis will be needed to measure the "worst-case" market power of an adversarial firm, which is an area left for future work.

We end this discussion with a note on supermodularity of market power. When market power is supermodular, it suggests that there is an incentive for generators to collude and form large firms. In fact, previous work in [11], [17] has suggested that there is always such an incentive. However, [11], [17] did not use power-flow equations in their study, and so we revisit this question here. Interestingly, it is indeed the case that, most of the time, market power is supermodular. However, this is not always the case, e.g., for the IEEE 39-bus system, supermodularity does not hold for generators at nodes 31 and 32 when the line-flow limits are uniformly scaled down to 70% of their given values. Other examples can also be found. Thus, while it is often the case that firms have incentive to collude, this is not universally true.

VII. Concluding remarks

In this paper, we proposed a functional long-term market power measure, called the *transmission constrained network flow* index, that uses AC power flow equations to model the underlying physical power network. This measure unifies three directions within market power research – residual supply based measures, network flow based measures, and minimal generation based measures. Additionally, to the best of our knowledge, this is the first market power analysis where AC power flows are considered (as opposed to the DC-approximated linearization). Our results highlight that this distinction is of fundamental importance. That is, using the AC model as opposed to the DC model yields fundamentally different conclusions about market power. This highlights the fact that a pure economic analysis is not enough to accurately analyze market power in electricity markets – accurately modeling the power flow constraints is crucial.

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