Complexity and Economics: computational constraints may not matter

FEDERICO ECHENIQUE
Caltech
and
DANIEL GOLOVIN
Caltech
and
ADAM WIERMAN
Caltech

Recent results in complexity theory suggest that various economic theories require agents to solve intractable problems. However, such results assume the agents are optimizing explicit utility functions, whereas the economic theories merely assume the agents are *rational*, where rational behavior is defined via some optimization problem. Might making rational choices be easier than solving the corresponding optimization problem? For at least one major economic theory, the theory of the consumer, we find this is indeed the case.

Categories and Subject Descriptors: F.2.0 [**Theory of Computation**]: analysis of algorithms and problem complexity–*general*; J.4 [**Social and Behavioral Sciences**]: Economics

General Terms: Economics, Algorithms

Additional Key Words and Phrases: Rationalization, Bounded Rationality

---

Questions related to the computational complexity of economic models have been a driving force inspiring research at the intersection of computer science and economics. Over the last decade, many traditional economic models have been examined with the goal of understanding the computational requirements placed on the agents involved. For many models, this question has now been (partially) resolved. For example computing Nash equilibria is now known to be hard [Daskalakis et al. 2009; Etessami and Yannakakis 2010] even for 2-player games [Chen et al. 2009]. Computing Walrasian equilibria is also hard [Chen et al. 2009; Vazirani and Yannakakis 2010].

Arguably, the most basic hardness result is for the theory of a single consumer seeking to maximize utility. The NP hardness of the *theory of the consumer* follows directly from the hardness of integer programming ([Gilboa et al. 2010] is a recent paper emphasizing this fact). Many computer scientists regard such hardness results as strong critiques of the applicability of classic economic models. For example:

> "If an equilibrium is not efficiently computable, much of its credibility as a prediction of the behavior of rational agents is lost"
> – Christos Papadimitriou [Nisan et al. 2007]

> "If your laptop cannot find it, neither can the market"
> – quote attributed to Kamal Jain [Nisan et al. 2007]

Authors' email addresses: fede@caltech.edu, dgolovin@caltech.edu, adamw@caltech.edu.

The critique claims that either these computationally–hard models should be rejected, or some suitable justification must be found for keeping them in spite of their worst-case hardness.

Economists have, in general, been slow to accept such critiques of these classic models. We posit that this is due to a differing view of the purpose of economic models, and that this view leads to a different notion of how computational issues should be incorporated. In particular, to an economist, models are not supposed to be literally true or correct; but rather provide tools for thinking about data. Thus, economic theories have an 'as-if' nature: economists postulate a model, and claim the observable variables behave *as if* the model were true. Given data from an observed phenomenon, we want to know if the theory can explain the data, i.e., if the data is consistent with an instance of the theory. If it is inconsistent, then the theory is refuted, or falsified.

Empirics and complexity. [Echenique et al. 2010] study the computational complexity of the theory of the consumer from an empirical 'as-if' perspective. The theory of the consumer is NP-hard because integer programming is NP-hard. Given data on consumption choices, we can ask if the data is compatible with utility maximization. Economists do not posit that consumers literally maximize a utility, but only that they act as if they do. Data take the form of a collection (x^k, p^k), $k = 1, \dots, K$, where $x^k \in \mathbb{Z}^d_+$ is a vector of purchases of d goods, and $p^k \in \mathbb{R}^d_+$ is a vector of prices for the d goods. The data is ***rationalizable*** if there is a monotonic function $u : \mathbb{Z}^d_+ \to \mathbb{R}$ such that x^k is the unique solution to

$$\begin{array}{ll} \max & u(x) \\ \text{s.t.} & p^k \cdot x \leq p^k \cdot x^k. \end{array}$$

The problem of rationalizable data in consumption theory (and other fields) is termed "revealed preference theory," and has a long tradition in economics. See [Varian 2006] for a survey.

Our main result is that if a data set is rationalizable, then it is rationalizable using a utility function that is easy to maximize in any budget set — easy in the sense that it can be done in strongly polynomial time. Informally the result can be stated as follows.

> A data set either falsifies the theory of utility maximizing behavior, or it is compatible with the joint hypotheses of utility maximization and computationally constrained consumers. *Thus computational constraints have no added empirical content.*

We prove our result by proposing a polynomial-time algorithm that computes a vector $x \in \mathbb{Z}^d_+$ in any budget set; that is in any set of the form $B(y, I) = \{y \in \mathbb{Z}^d_+ : p \cdot y \leq I\}$, for $p \in \mathbb{R}^d$ and $I > 0$. Such an algorithm computes a demand function for the consumer. Our algorithm has two crucial features. The first is that it replicates the consumer's choice at the observed data (x^k, p^k), $k = 1, \dots, K$. The second is that its choices maximize some monotonic utility function (we do not explicitly construct the utility, though). We refer the reader to [Echenique et al. 2010] for the details of the algorithm and the proof that it has the stated properties.

Discussion. The theory of the consumer appears to assume that economic agents solve NP-hard problems. One may question the theory on the grounds that this assumption is unrealistic. Indeed, computer scientists question classical economic theories on these grounds, as the quotes above illustrate. We have argued that this is not right because the realism of a theory has to be judged relative to what can be observed.

We show that the notion that economic agents have limited computational resources adds no empirical content to the theory of utility maximization. A data set of observed consumption at different budgets is either in contradiction with the hypothesis of utility maximization, or it can be explained using a utility function that is easy to maximize.

Our paper is not a critique of the literature on complexity and economics in general; rather, we take issue with the idea that worst-case hardness of a model implies that the model is flawed. We emphasize that the existing results on complexity are useful for understanding how economics can be applied in a normative and algorithmic way — for example, to engineer economic systems with desirable properties. We posit that while computer scientists tend to think 'algorithmically' about economic models, economists tend to think 'empirically' about the models. There is a need for considerations of computational complexity in both views.

In particular, an *algorithmic view* of economic models assumes that the model is fixed and literally true, and then proceeds to ask about the computational demands placed on the agents by the model. That is, it assumes that the agent is simply an implementation of the model and asks whether the agent can efficiently compute its decisions.

In contrast, an *empirical view* takes the model as a tool for thinking about reality. One does not presume agents literally follow the model, only that the model provides a way to explain the observed behavior. In this view, a model still loses credibility if the agents must solve computationally hard problems; however, worst-case complexity is no longer the relevant concept. Instead, the question is whether data from an observed phenomenon can always be explained by the theory with the additional constraint that agents are not required to solve computationally hard problems. This is the case with the theory of the consumer. On the other hand, we expect complexity to matter empirically for other economic models. When that is the case, one would want to characterize the added empirical consequences of assuming that economic agents do not solve hard problems.

REFERENCES

CHEN, X., DAI, D., DU, Y., AND TENG, S.-H. 2009. Settling the complexity of arrow-debreu equilibria in markets with additively separable utilities. In *FOCS '09: Proceedings of the 2009 50th Annual IEEE Symposium on Foundations of Computer Science*. IEEE Computer Society, Washington, DC, USA, 273–282.

CHEN, X., DENG, X., AND TENG, S.-H. 2009. Settling the complexity of computing two-player Nash equilibria. *J. ACM 56,* 3, 1–57.

DASKALAKIS, C., GOLDBERG, P. W., AND PAPADIMITRIOU, C. H. 2009. The complexity of computing a Nash equilibrium. *SIAM J. Comput. 39,* 1, 195–259.

ECHENIQUE, F., GOLOVIN, D., AND WIERMAN, A. 2010. A revealed preference approach to computational complexity in economics. Caltech SSWP 1333.

ETESSAMI, K. AND YANNAKAKIS, M. 2010. On the complexity of Nash equilibria and other fixed points. *SIAM J. Comput. 39,* 6, 2531–2597.

GILBOA, I., POSTLEWAITE, A., AND SCHMEIDLER, D. 2010. The complexity of the consumer problem and mental accounting. Mimeo: Tel-Aviv University.

NISAN, N., ROUGHGARDEN, T., TARDOS, E., AND VAZIRANI, V. V., Eds. 2007. *Algorithmic Game Theory*. Cambridge University Press, New York, NY, USA.

VARIAN, H. R. 2006. Revealed preference. In *Samuelsonian economics and the twenty-first century*, M. Szenberg, L. Ramrattan, and A. Gottesman, Eds. Oxford University Press, USA, 99–115.

VAZIRANI, V. V. AND YANNAKAKIS, M. 2010. Market equilibrium under separable, piecewise-linear, concave utilities. In *Proc. Innovations in Computer Science*. Beijing, China, 156–165.